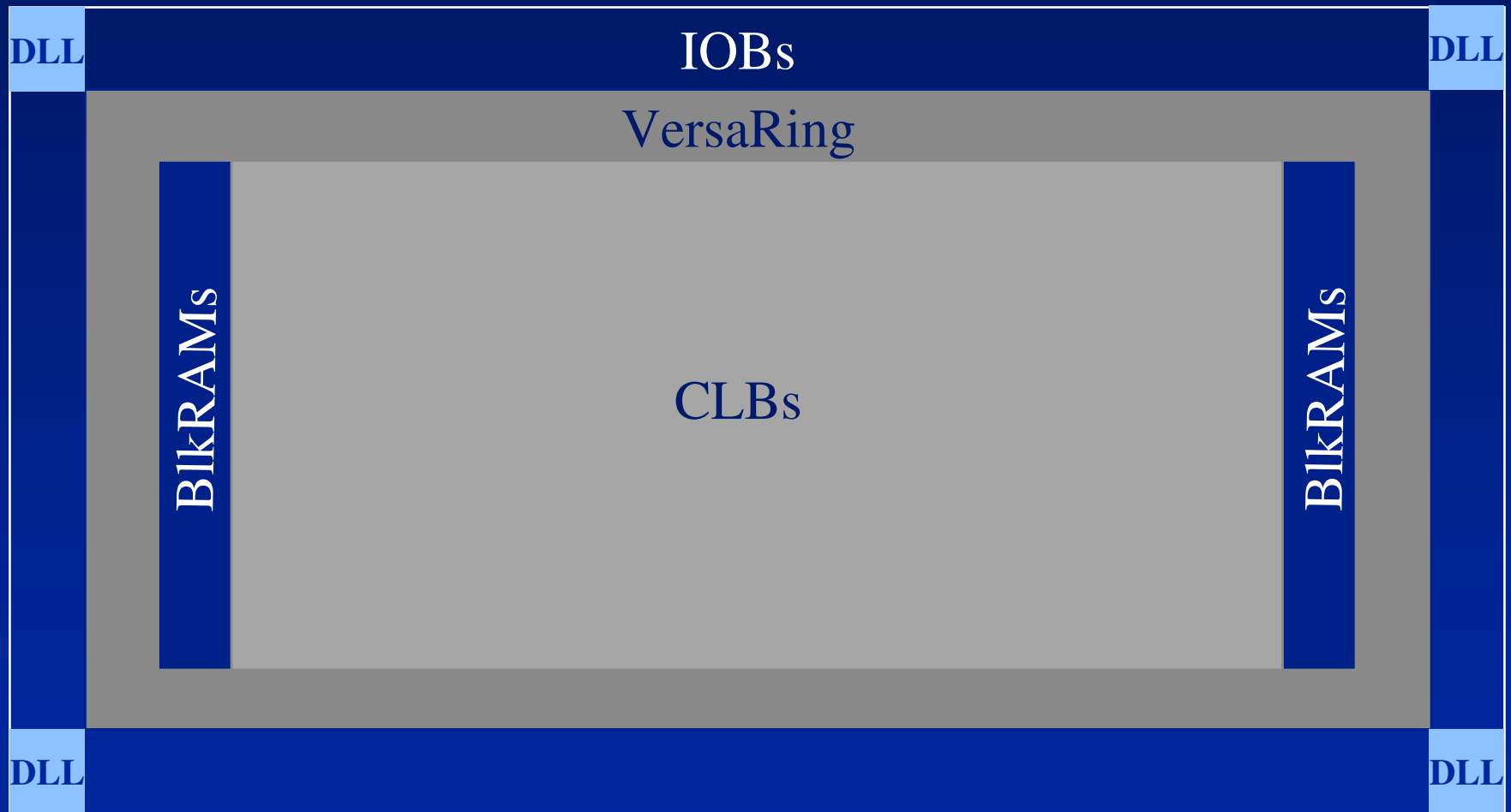


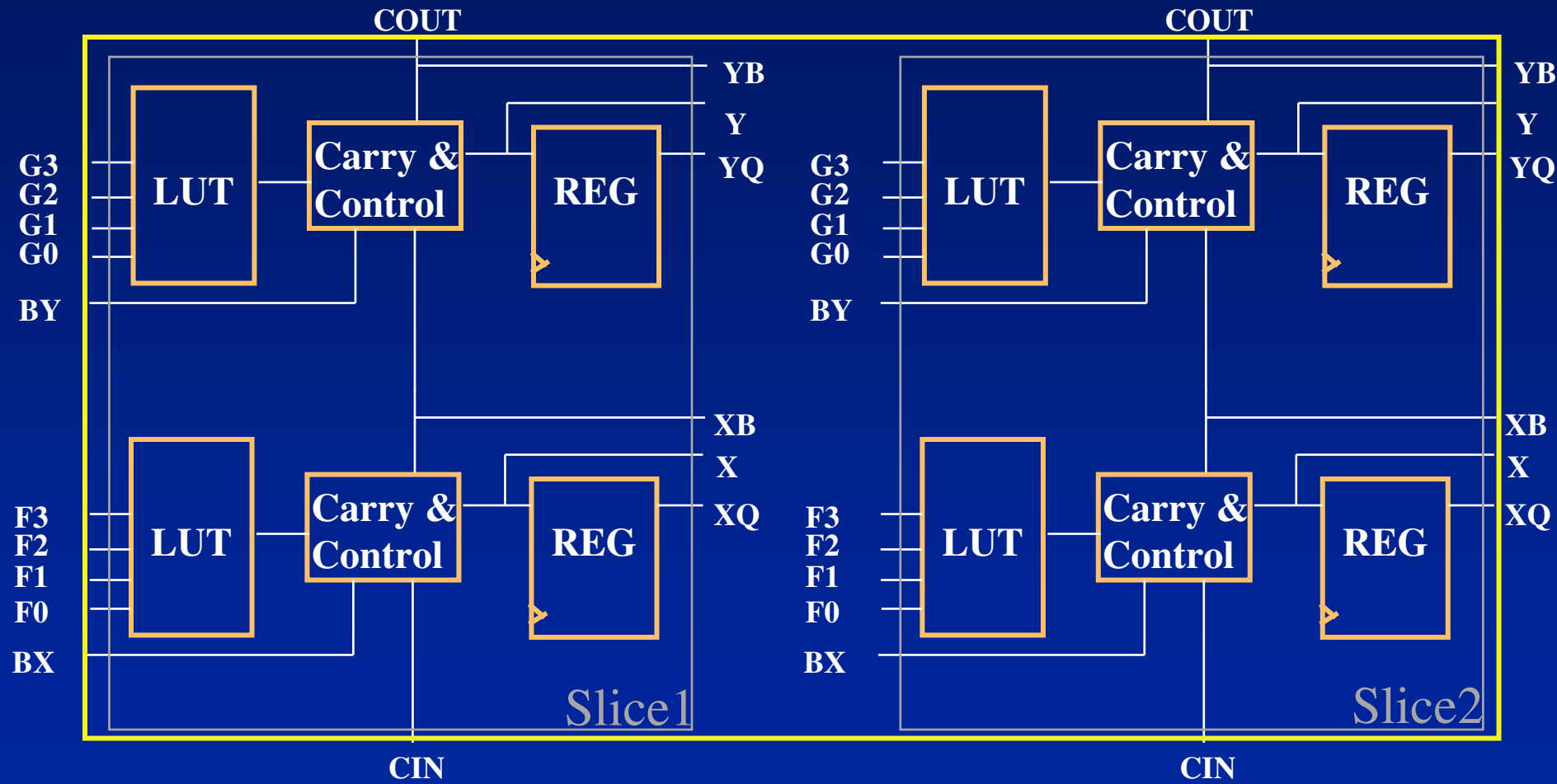


Virtex™ Architecture

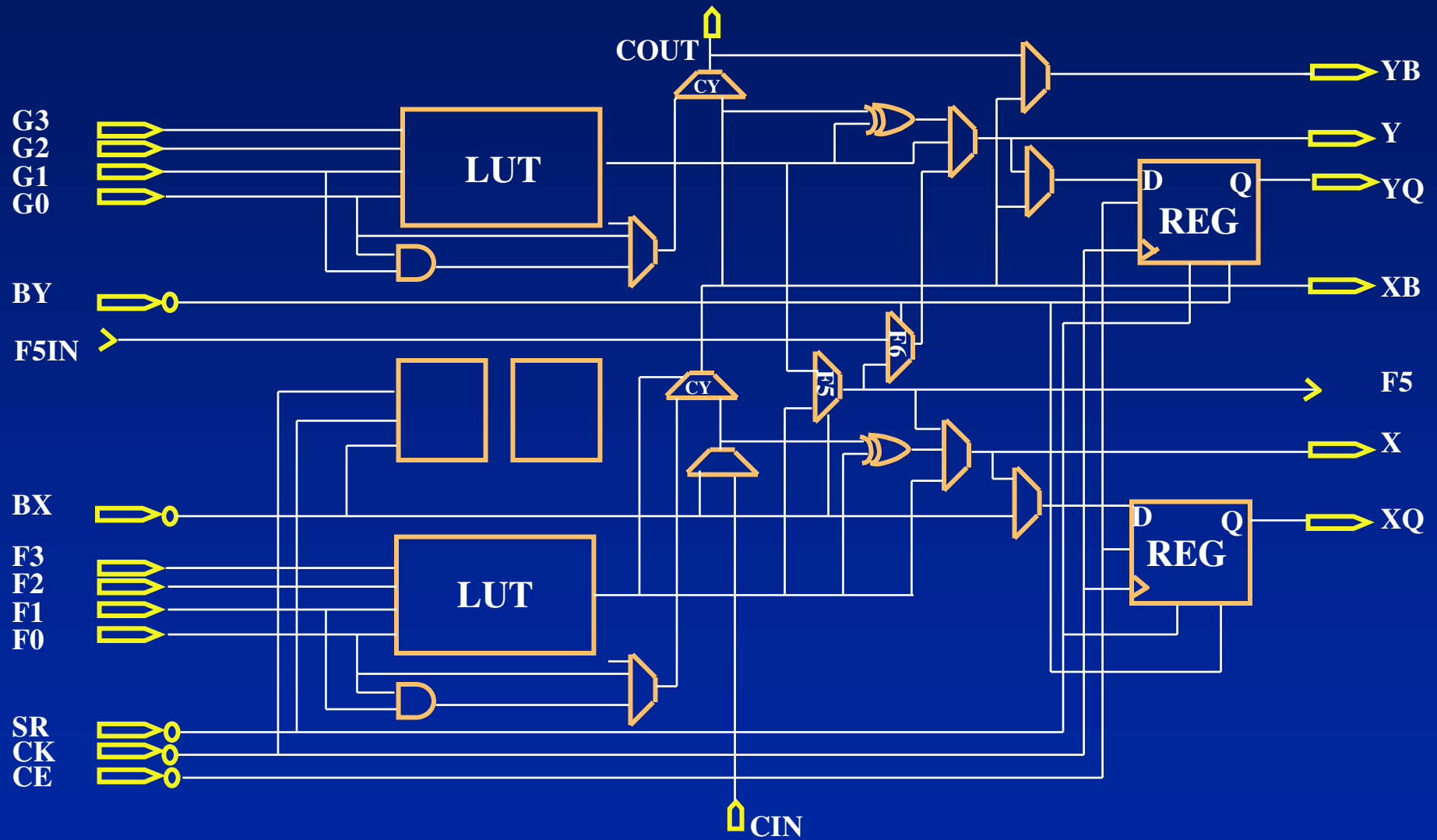
VIRTEX™ Architecture



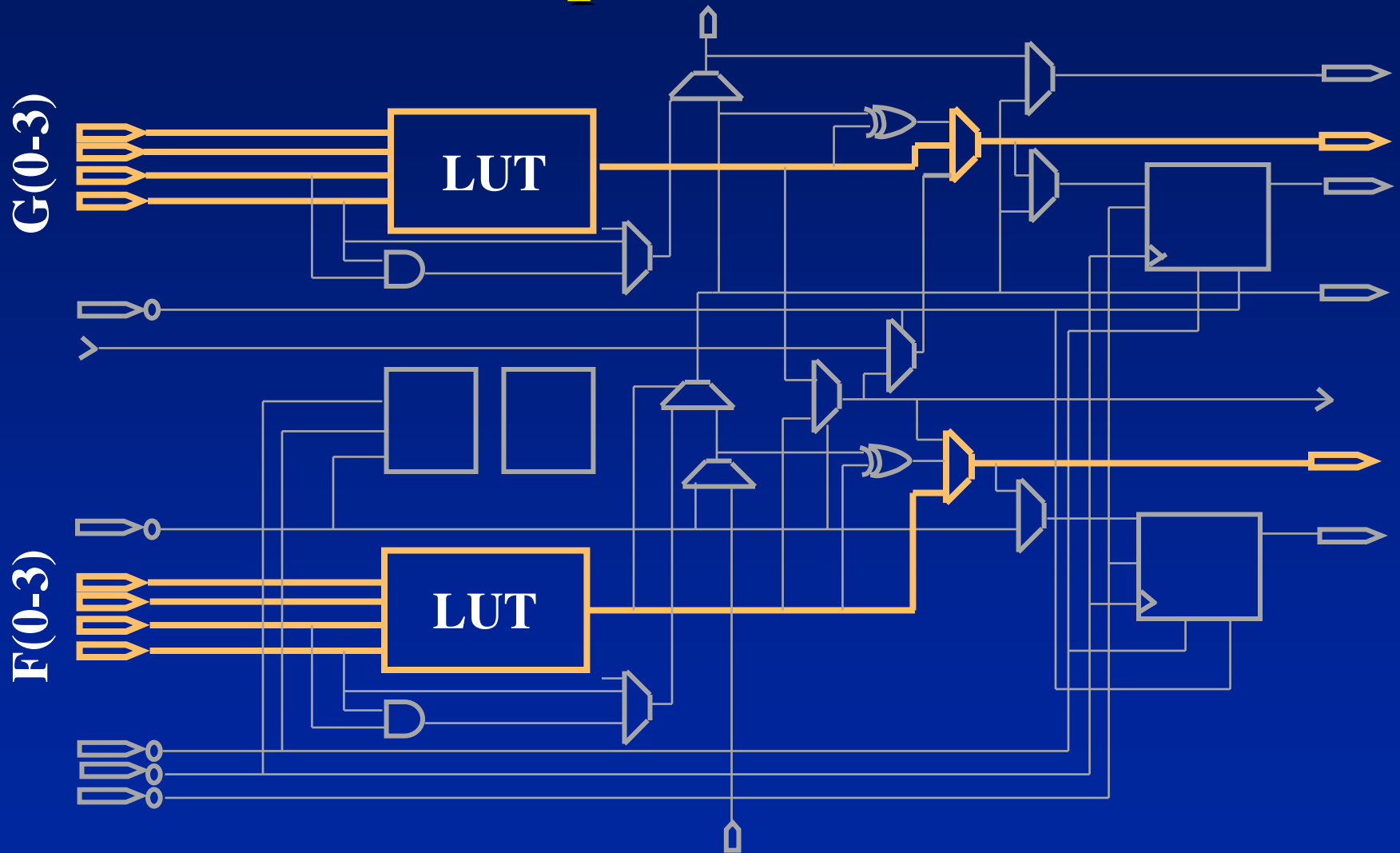
VIRTEX™ CLB



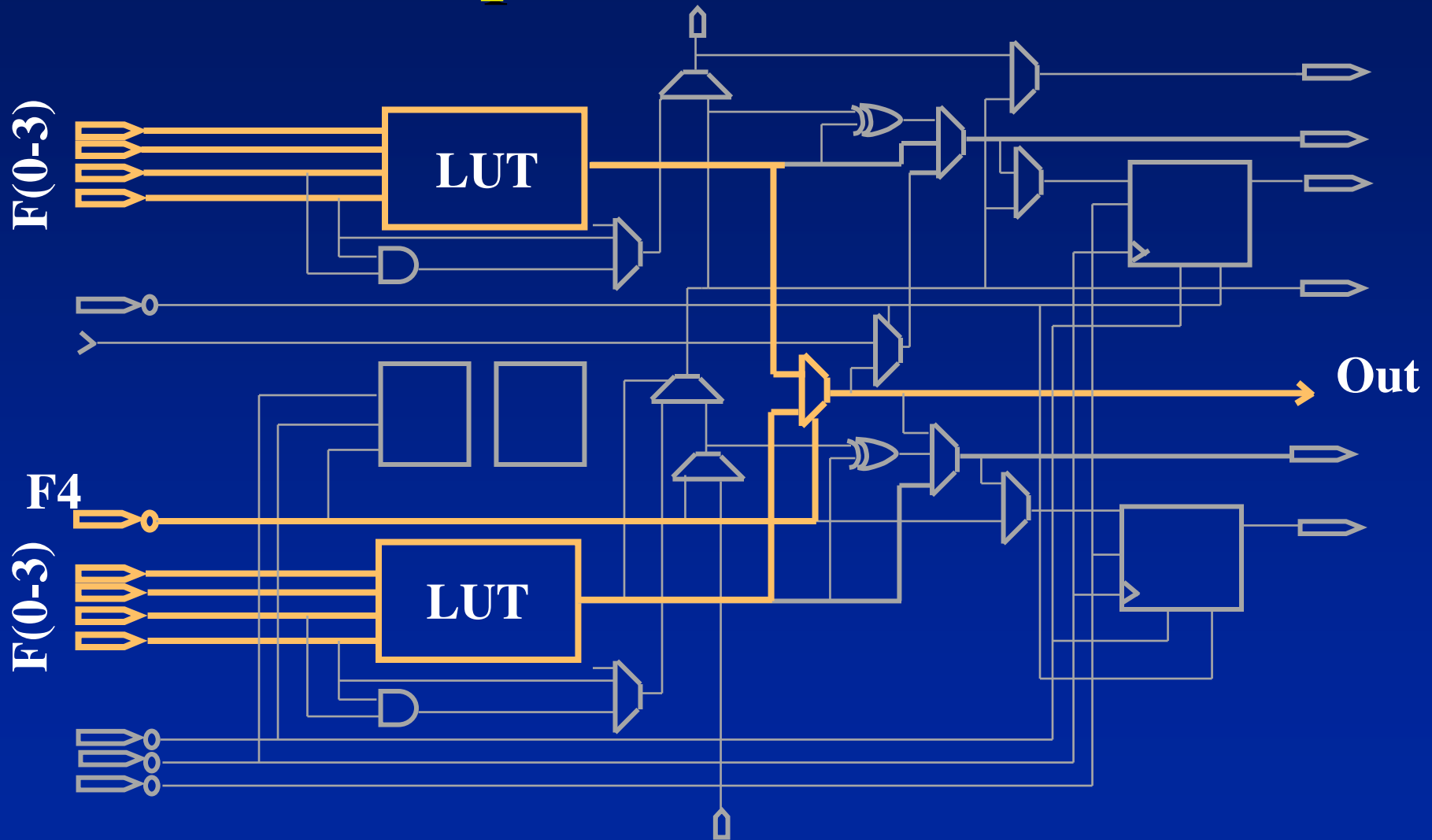
VIRTEX™ SLICE



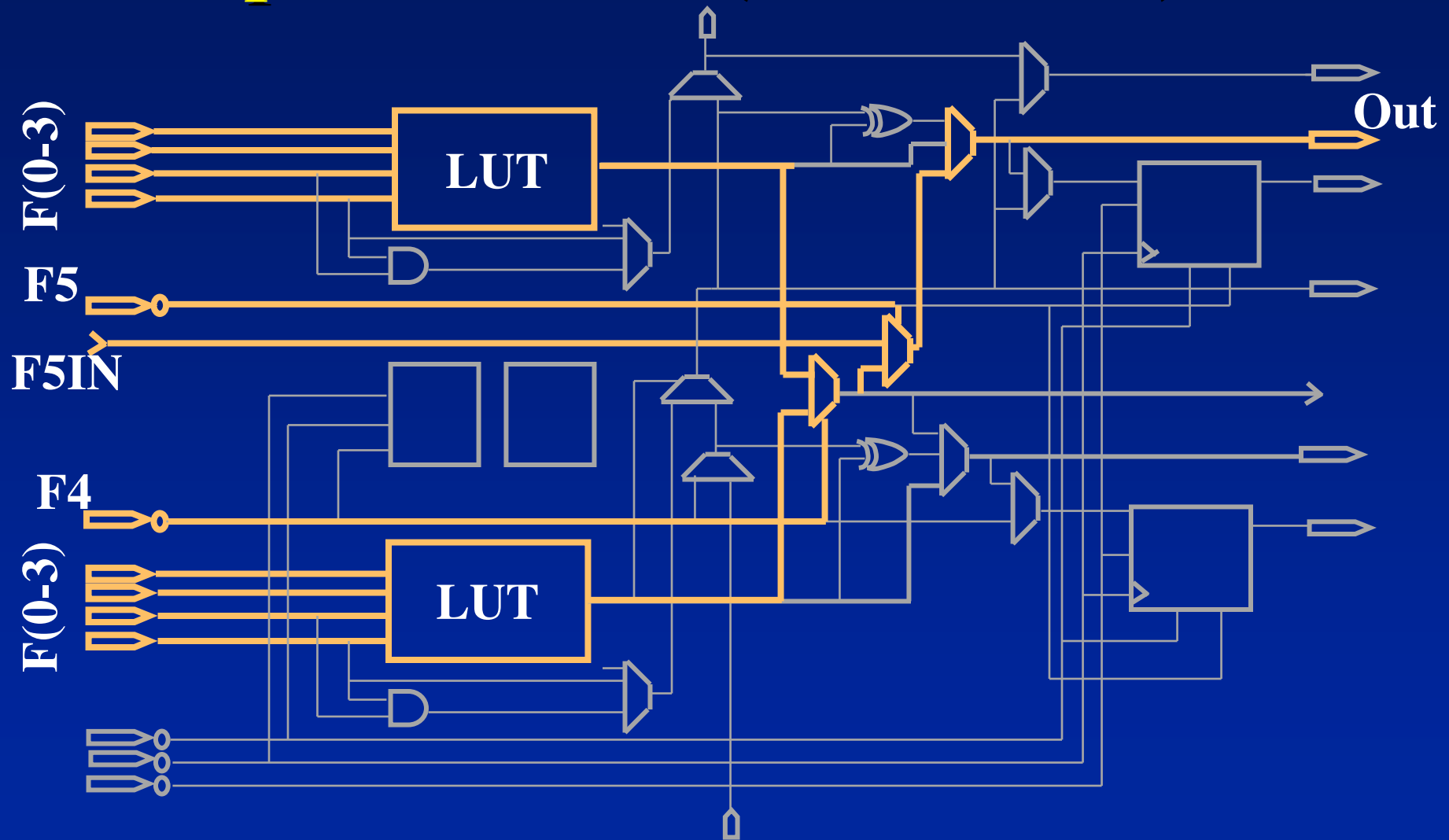
Two 4-input functions



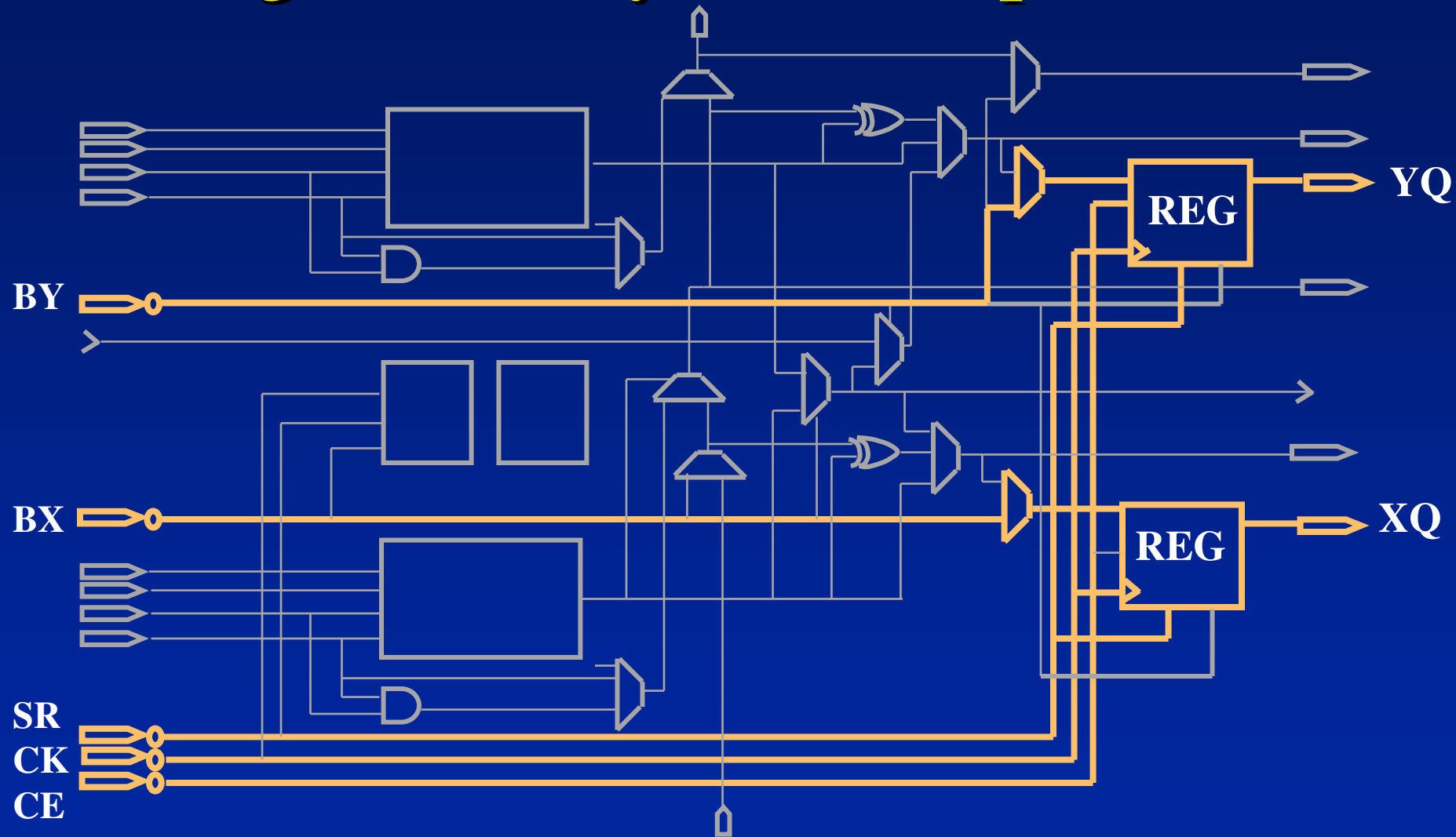
5-input function



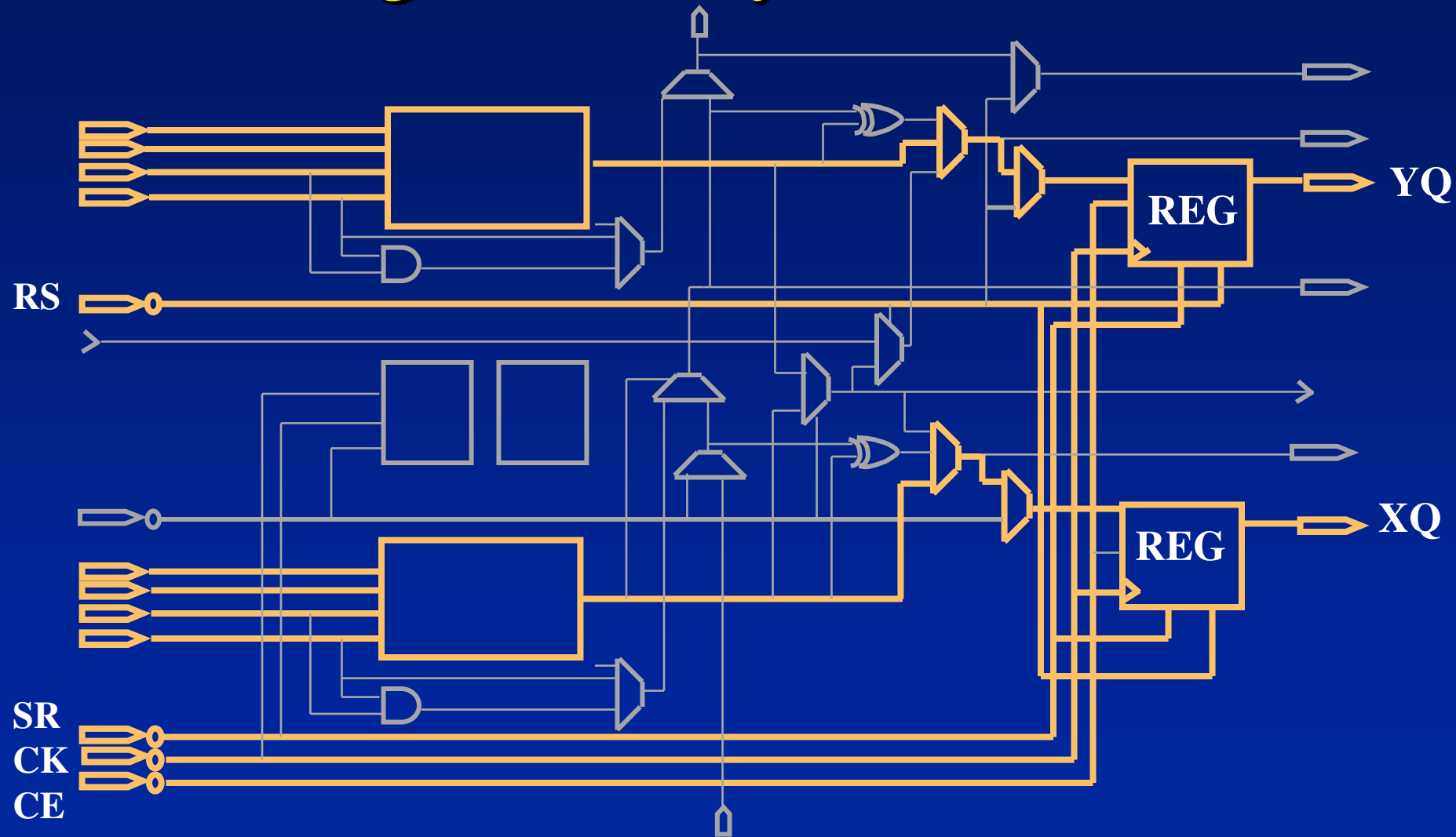
6-input function (half shown)



2 reg driven by direct inputs



2 reg driven by LUTs



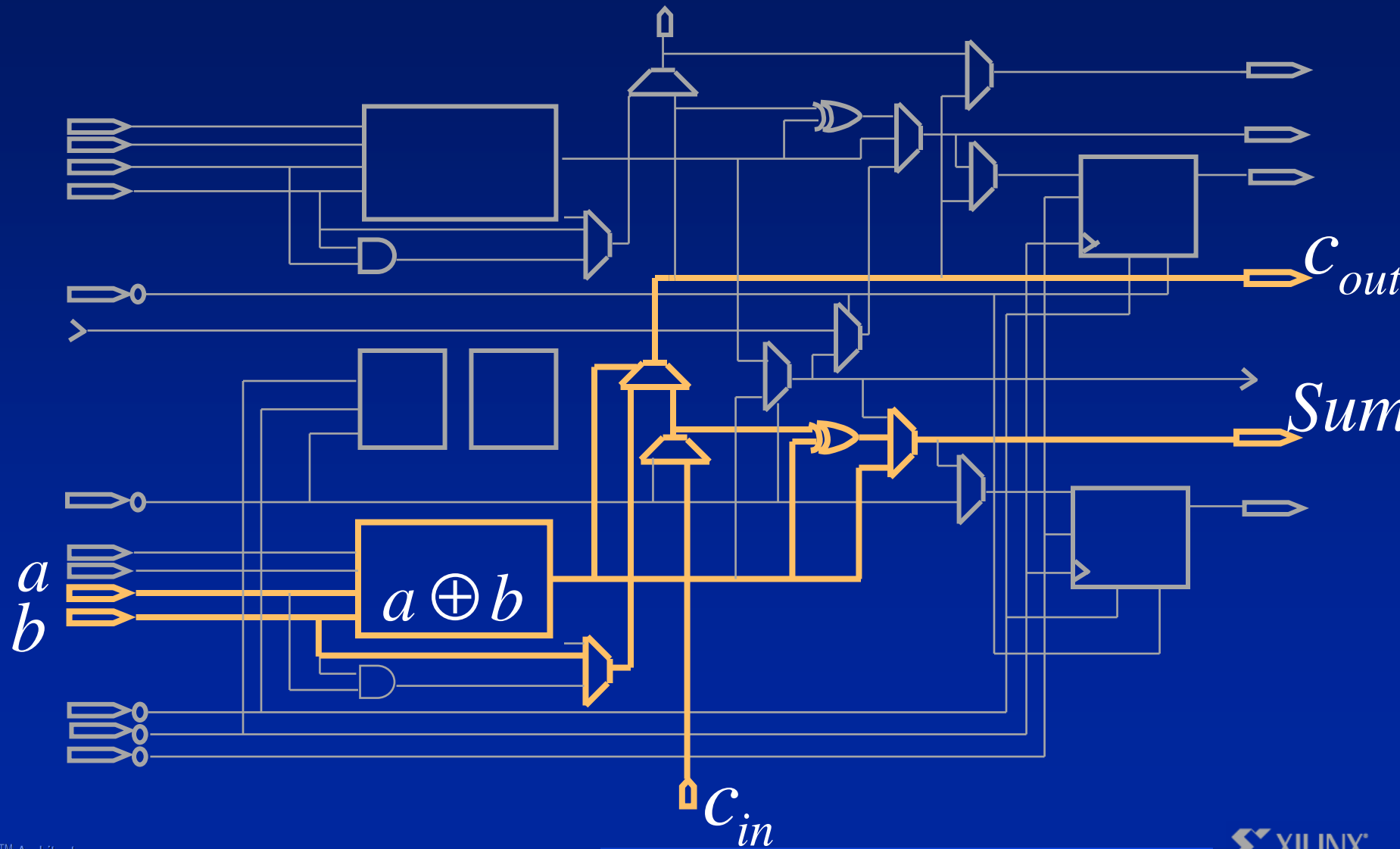
1-bit Full Adder

- ◆ Adding a and b; carry in C_{in}

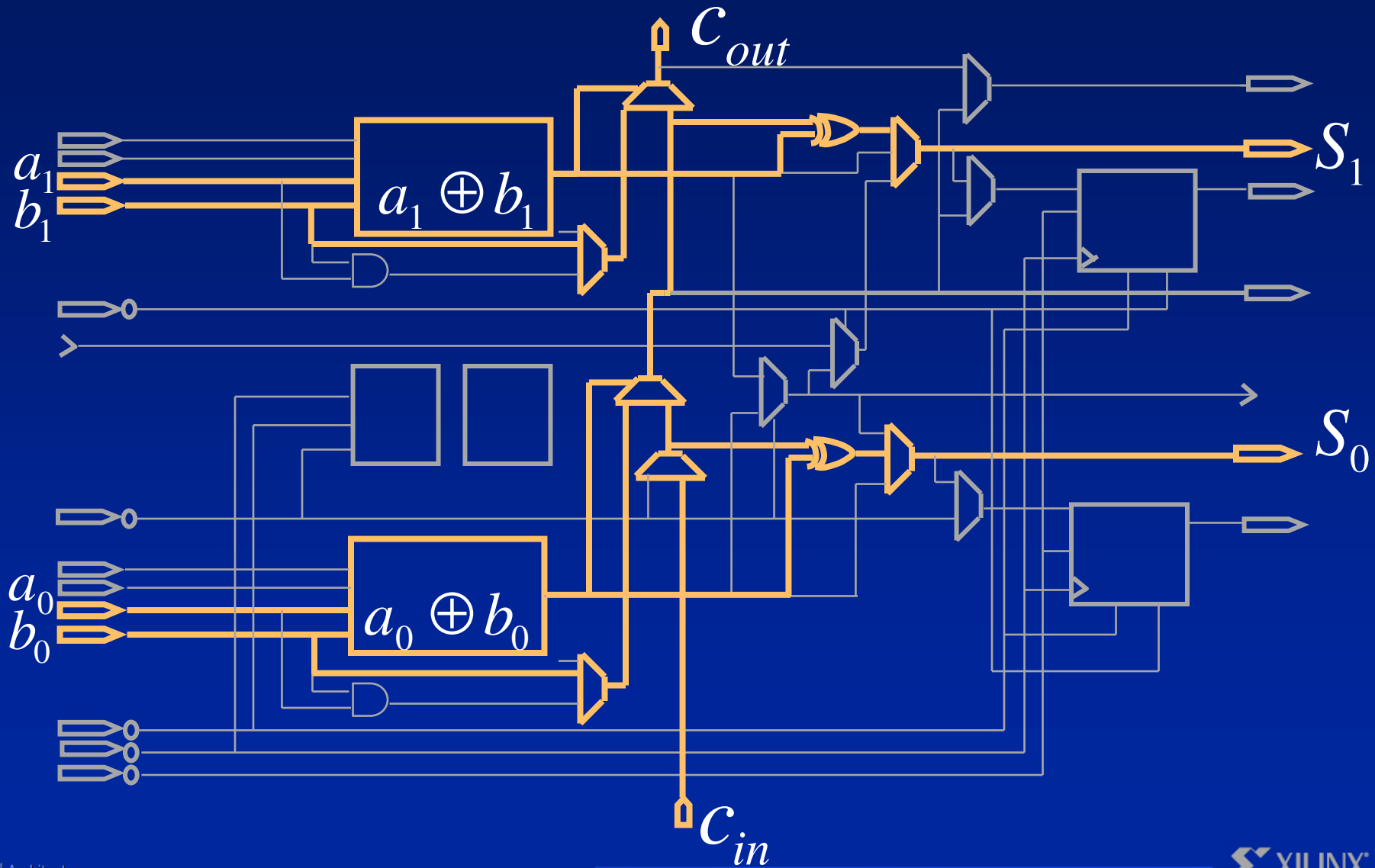
- ◆ $Sum = (a \oplus b) \oplus c_{in}$

- ◆
$$c_{out} = ab + ac_{in} + bc_{in}$$
$$= (a \oplus b) \cdot c_{in} + \overline{(a \oplus b)} \cdot b$$

1-bit full adder

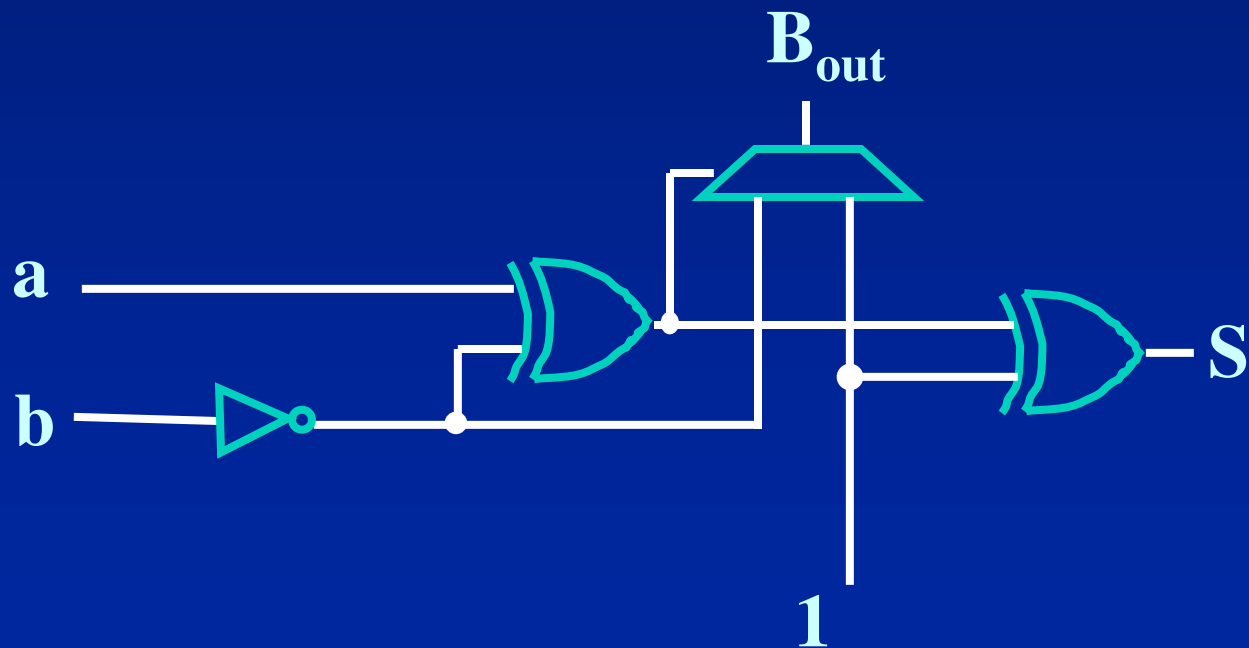


2-bit adder cascaded

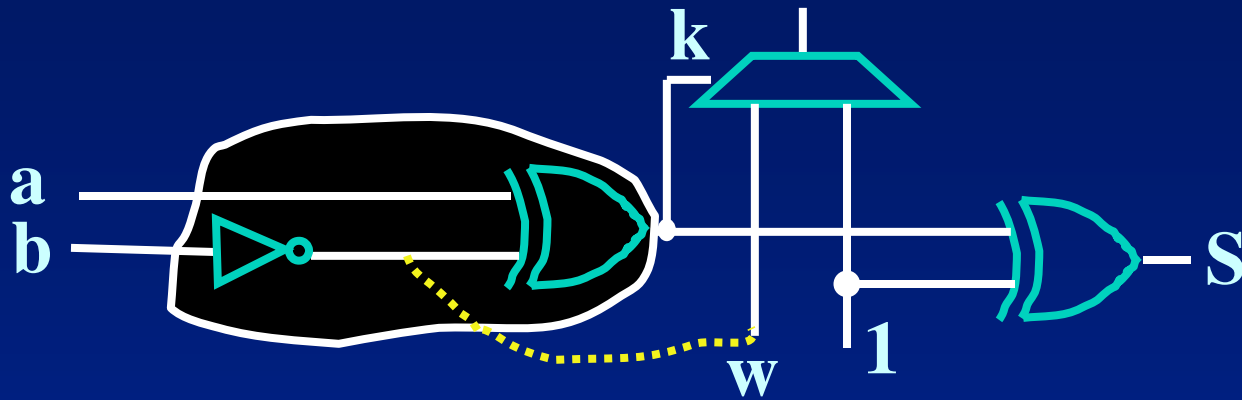


1-bit Subtractor

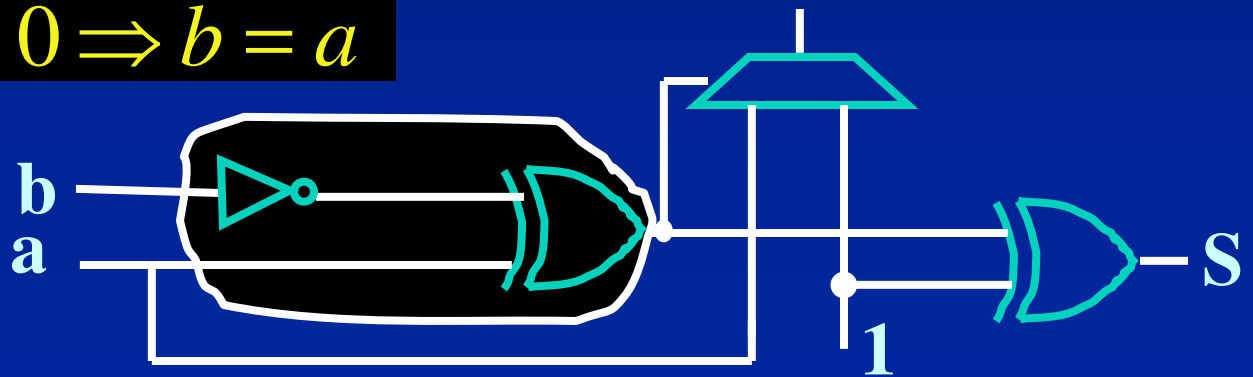
$$a - b = a + \bar{b} + 1$$



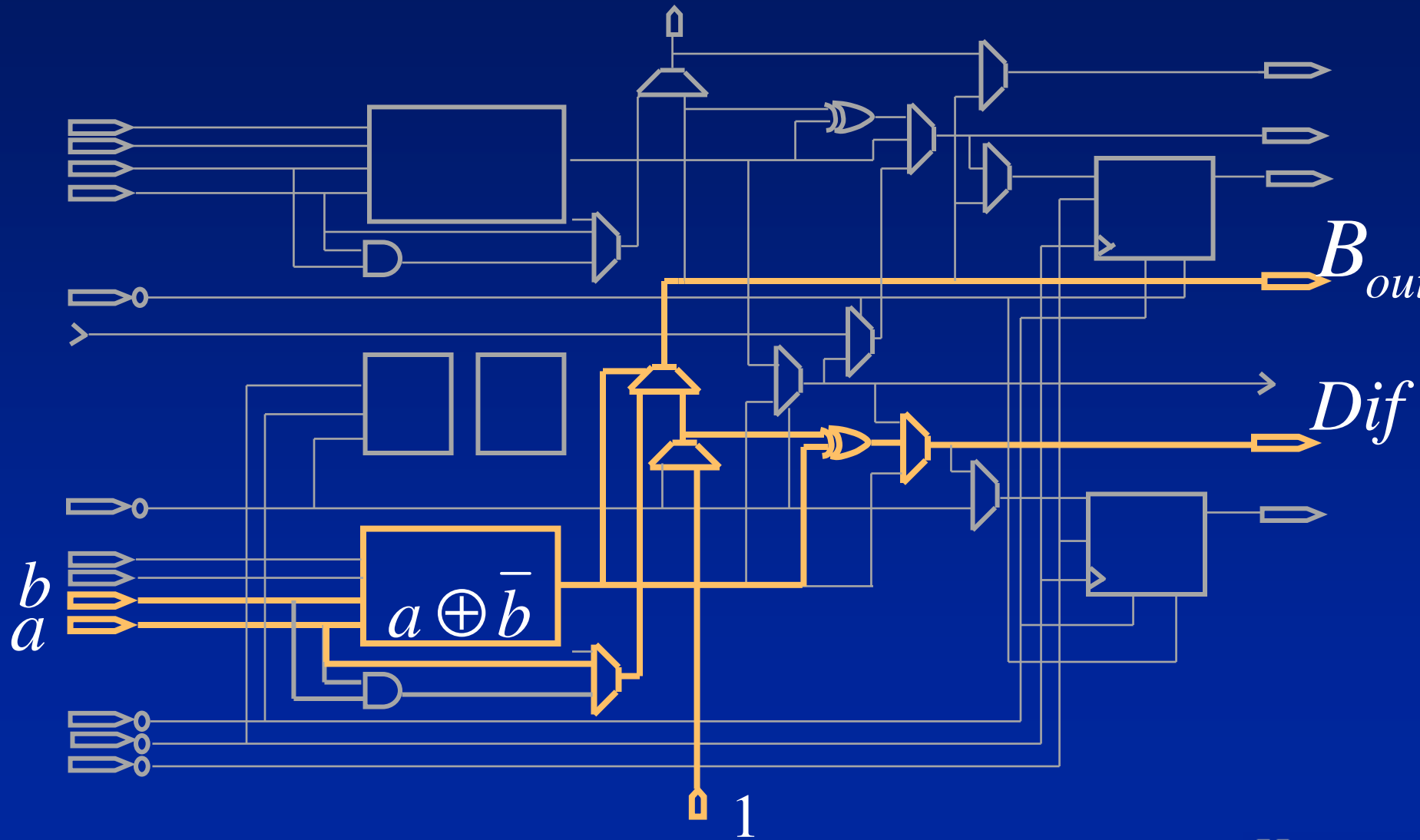
1-bit Subtractor



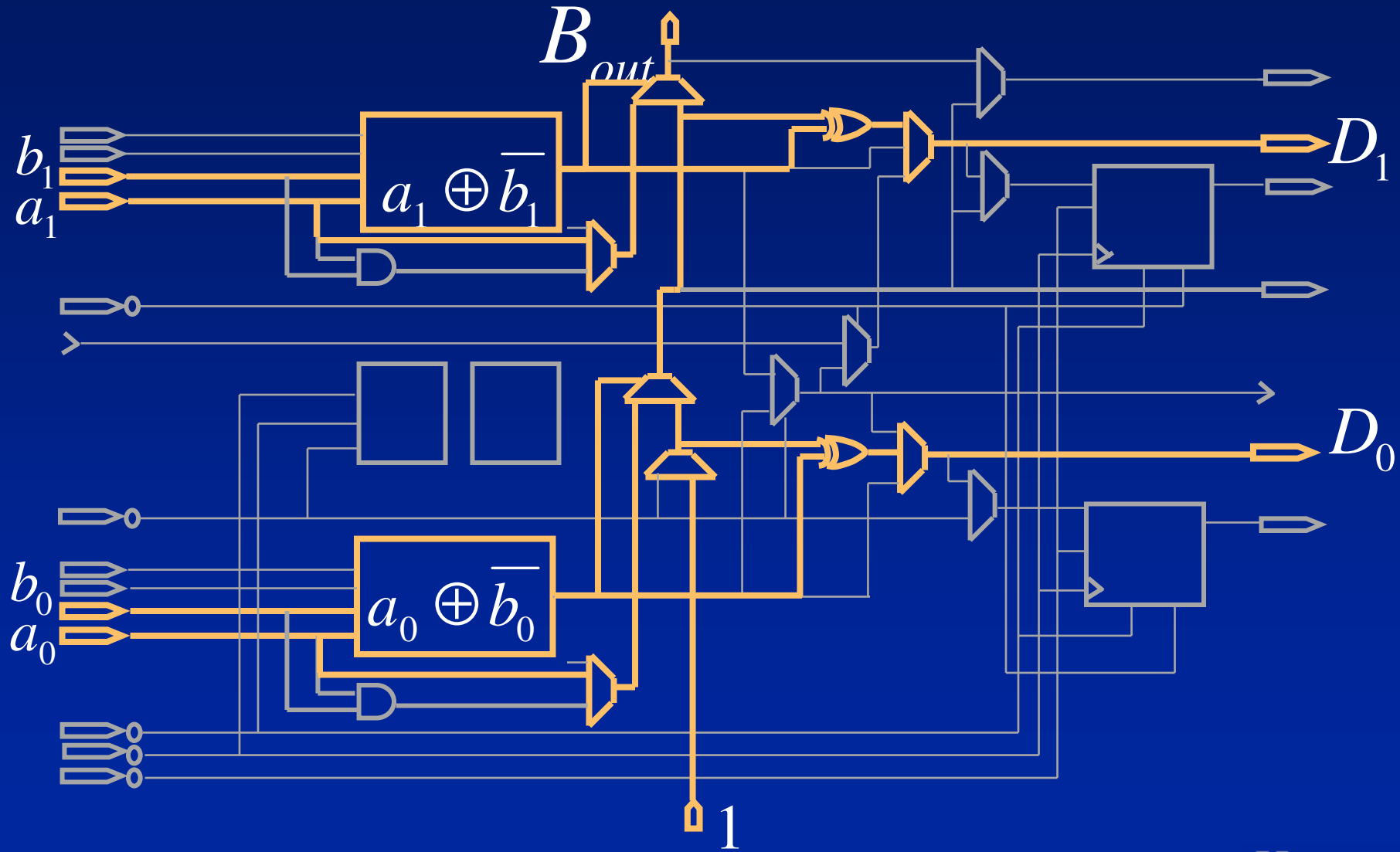
when ($k == 0$) reqd. $w = \bar{b}$
 $k = 0 \Rightarrow a \oplus \bar{b} = 0 \Rightarrow \bar{b} = a$



1-bit subtractor

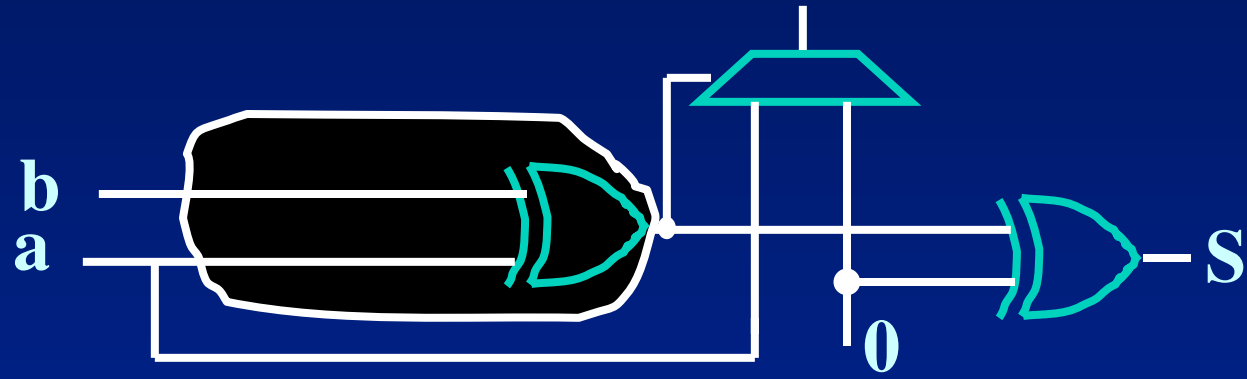


2-bit subtractor

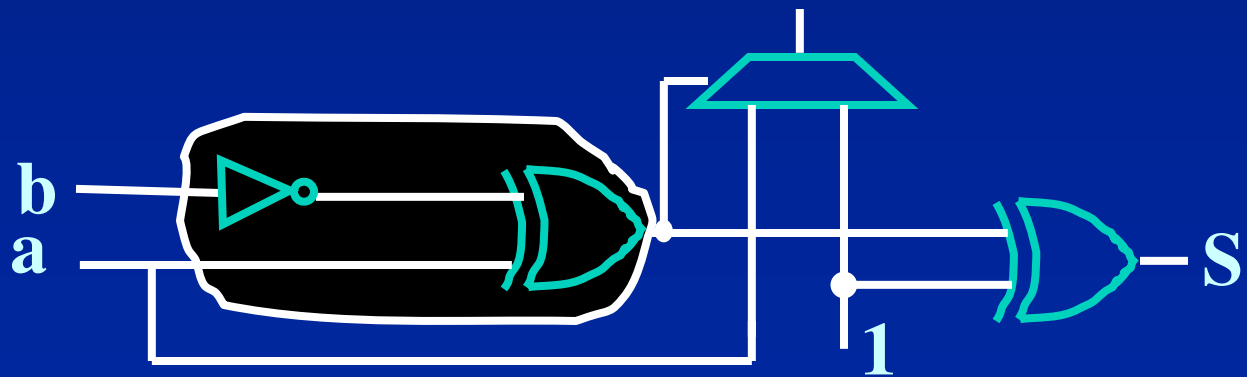


1-bit add-sub

$$sa = 0 \Rightarrow a + b$$



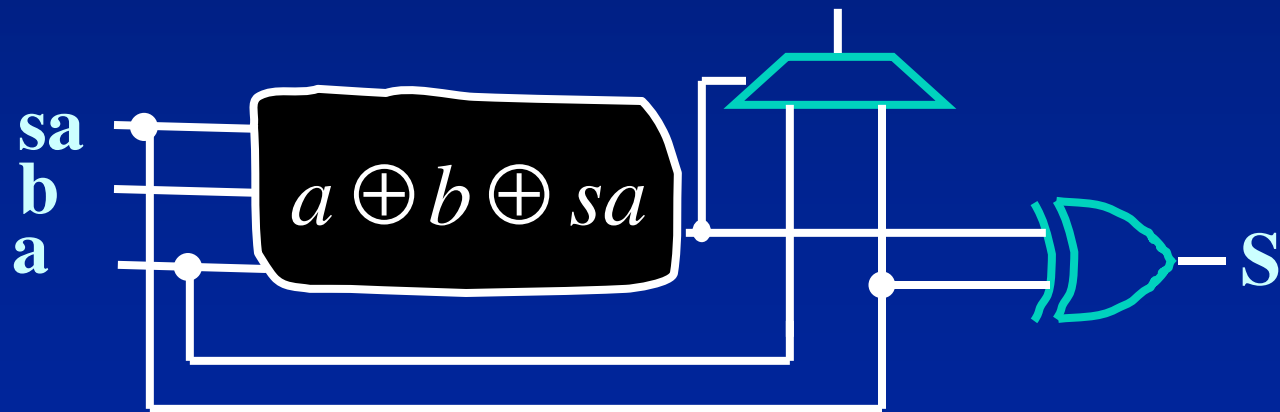
$$sa = 1 \Rightarrow a - b$$



1-bit add-sub

$$sa = 0 \Rightarrow a + b$$

$$sa = 1 \Rightarrow a - b$$



Scaling Accumulator

$$Y = X + 2^{-1}Y$$

- ◆ X : 16-bit input
- ◆ Y : 17-bit output

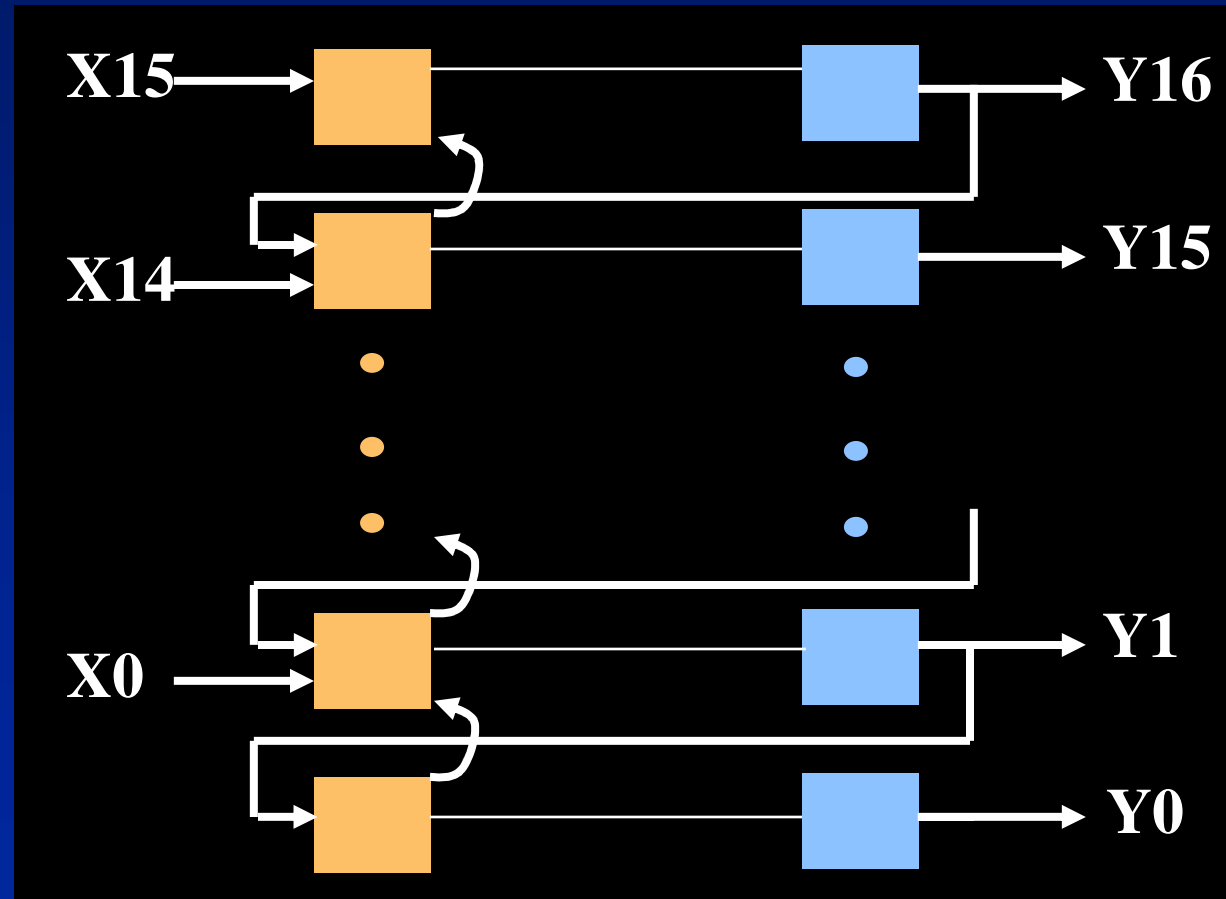
Scaling Accumulator

$$Y = X + 2^{-1} Y$$

 2-bit adder

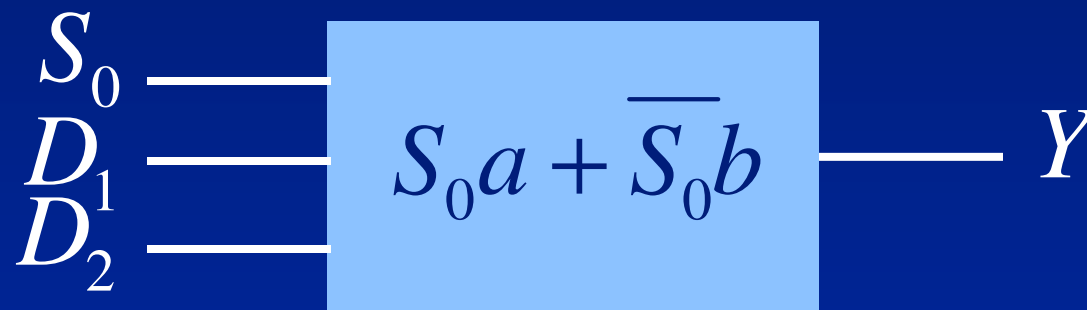
 Register

N-bit scaling acc:
N/2 slices or
N/4 CLBs



2:1 Multiplexer

$$Y = S_0 D_1 + \overline{S_0} D_2$$



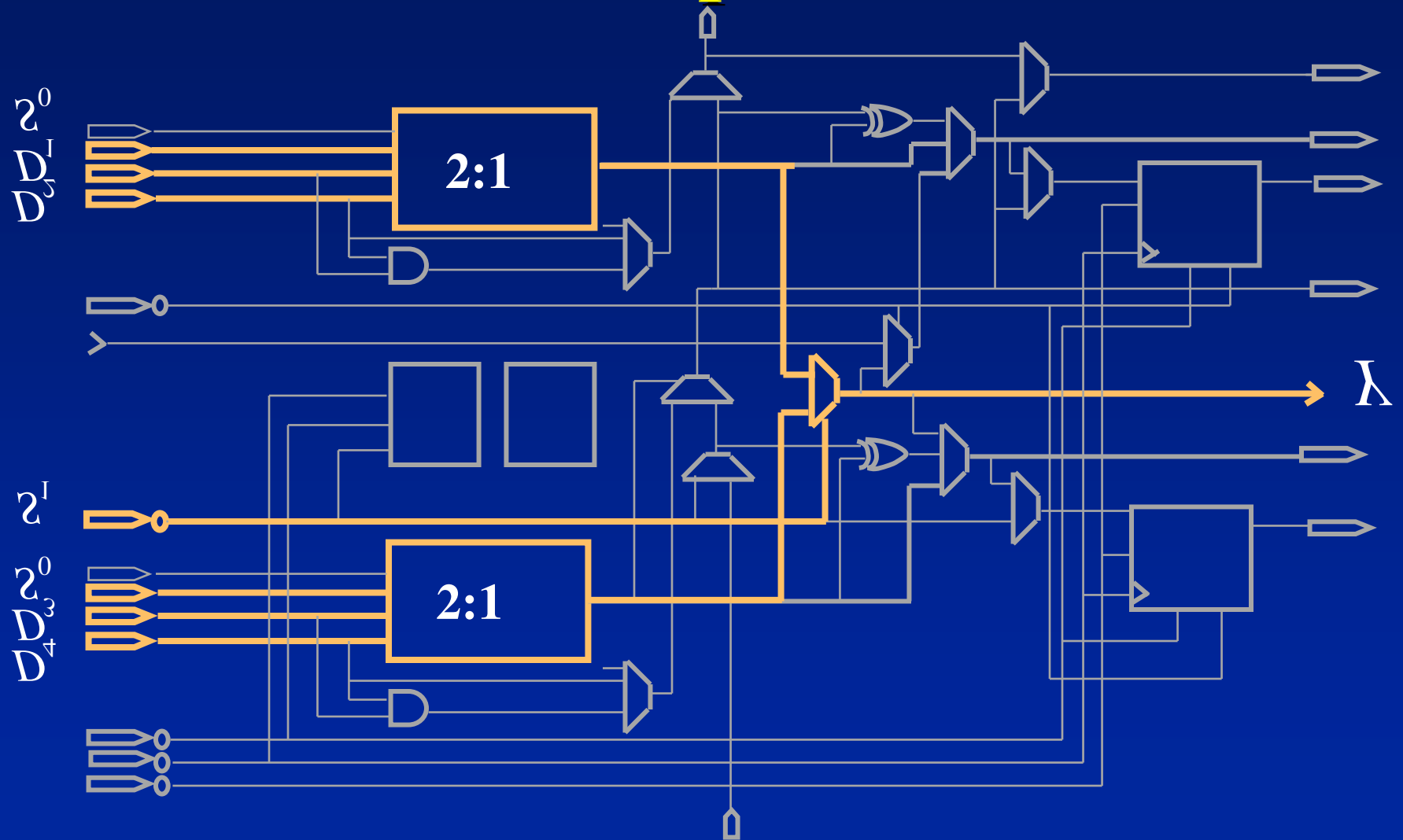
- ◆ Requires 1 LUT

4:1 Multiplexer

$$Y = \overline{S_0}\overline{S_1}D_1 + S_0\overline{S_1}D_2 + \overline{S_0}S_1D_3 + S_0S_1D_4$$

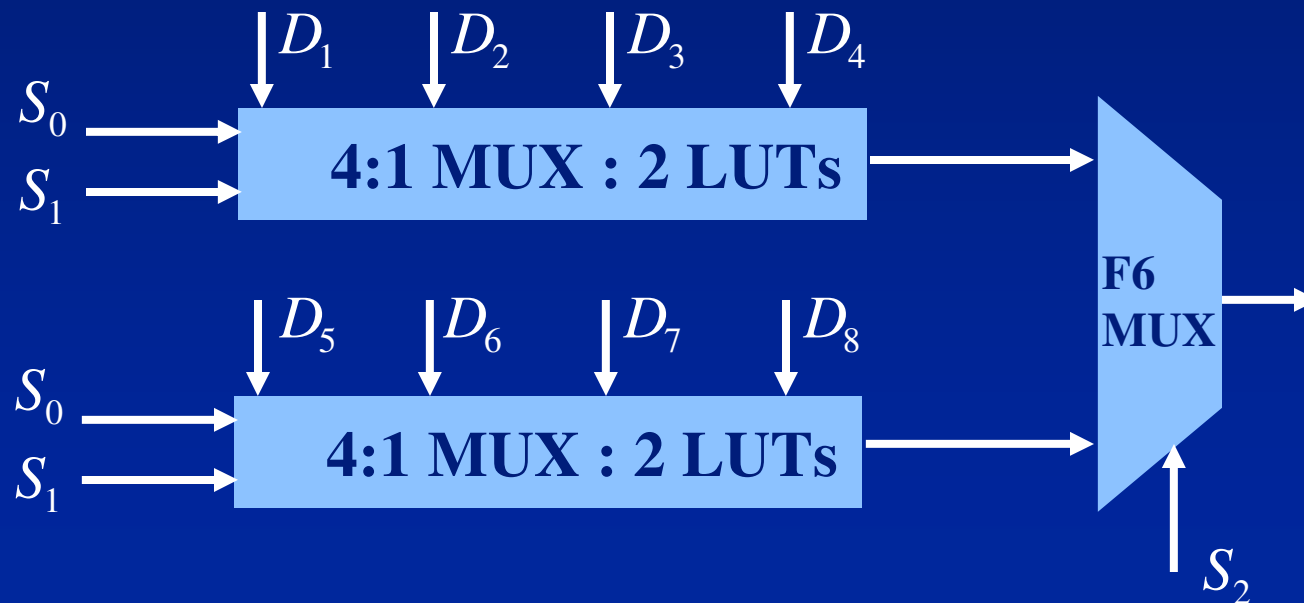
- ◆ 6 input function
- ◆ naïve implementation will require 4 LUTs
- ◆ exploit partitioning of variables
 - can implement using just 2 LUTs (1 slice)

4:1 Multiplexer



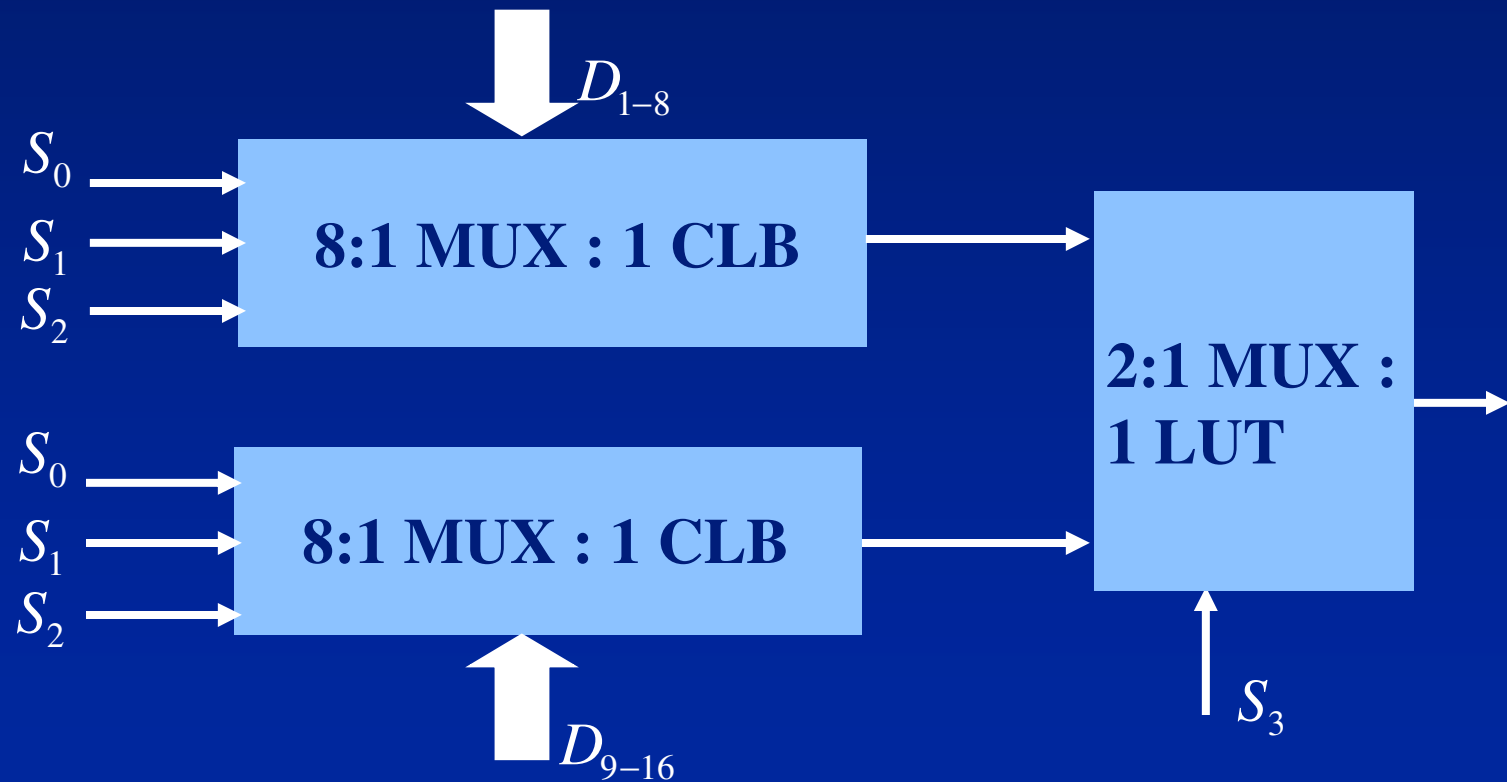
8:1 Multiplexer

- ◆ 11 input function
- ◆ can be implemented with 4 LUTs (1 CLB)

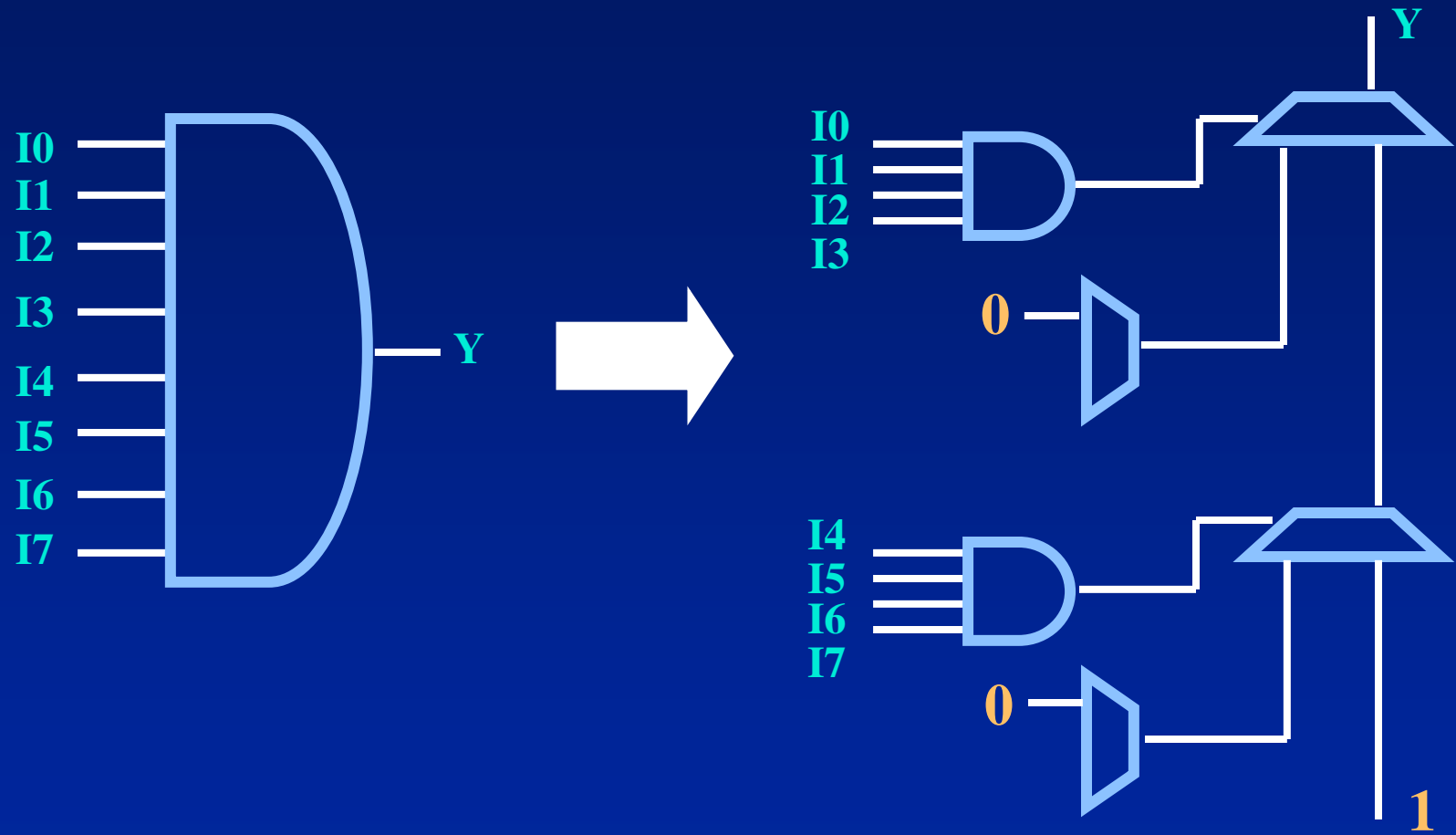


16:1 Multiplexer

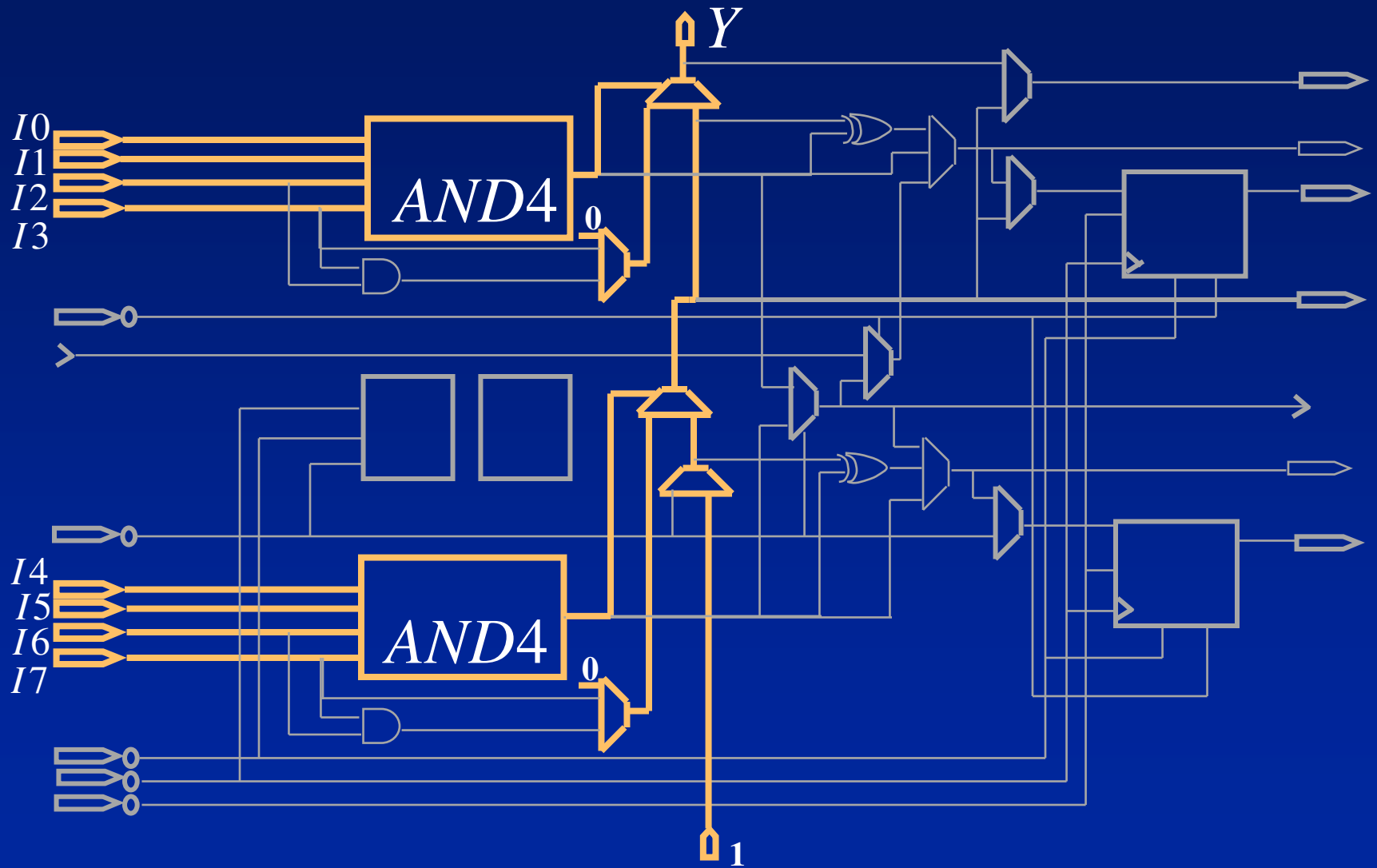
- ◆ Build larger muxes using [8:1] [4:1] [2:1] modules



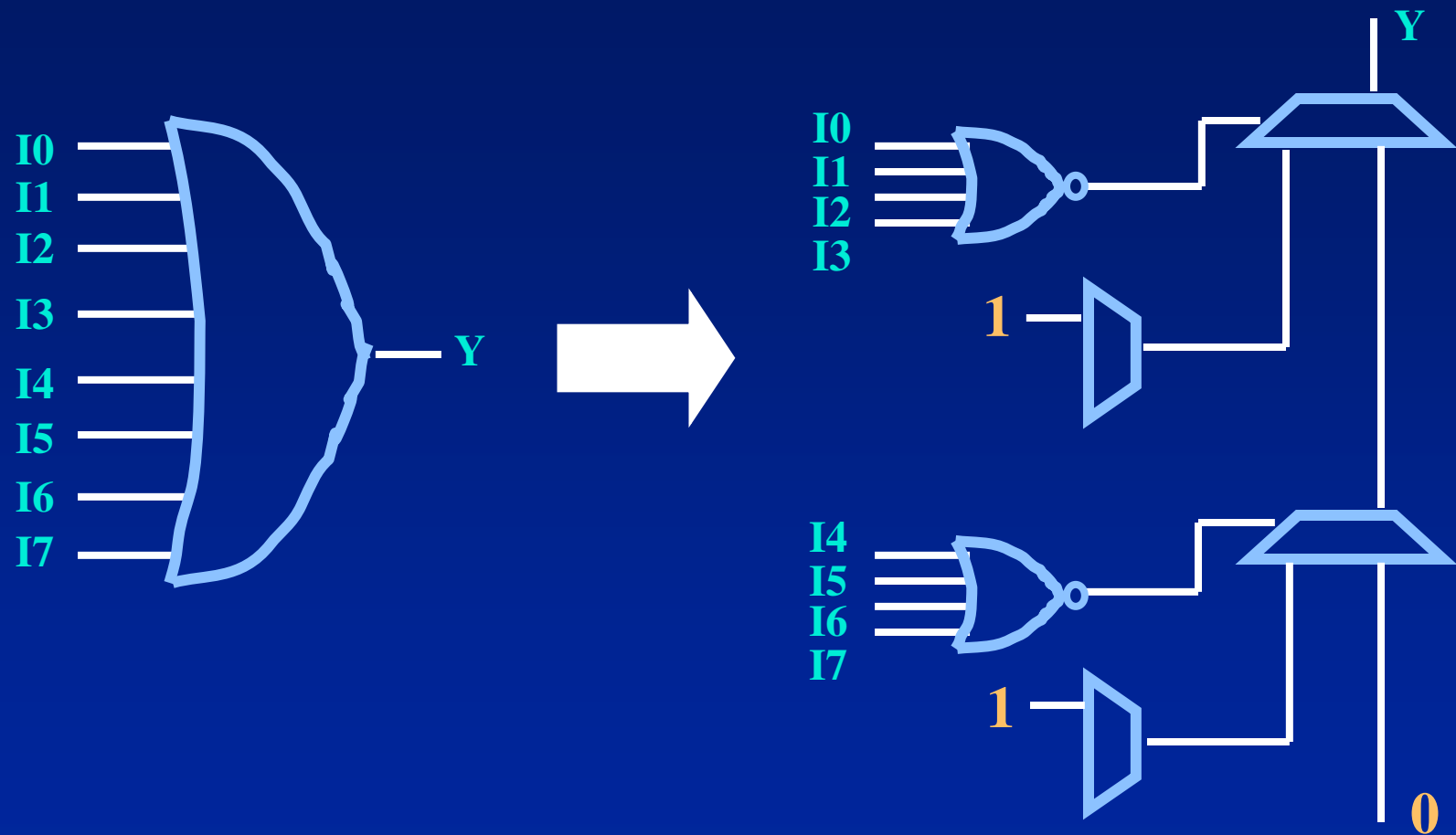
Wide-AND



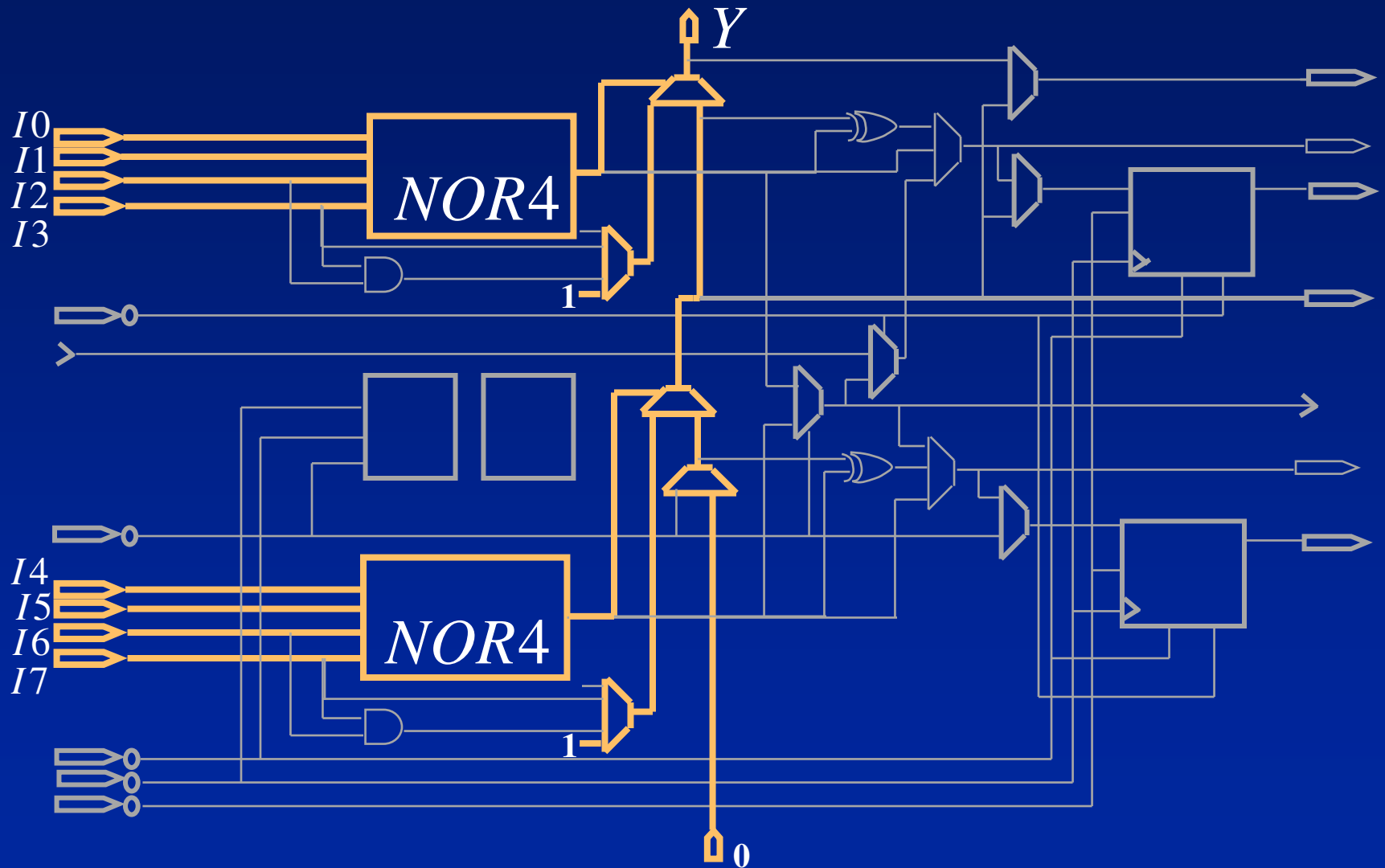
8-bit Wide-AND



Wide-OR



8-bit Wide-OR



Multiplier

$$Y = a \times b$$

- ◆ a : 16-bit multiplicand
- ◆ b : 16-bit multiplier
- ◆ Y : 32-bit output

Multiplier

$$\begin{array}{r} a_N a_{N-1} \dots a_2 a_1 a_0 \\ \times \quad b_M b_{M-1} \dots b_2 b_1 b_0 \\ \hline \end{array}$$

$$\begin{array}{r} a_N b_0 \dots a_2 b_0 a_1 b_0 a_0 b_0 \\ a_N b_1 \dots a_2 b_1 a_1 b_1 a_0 b_1 \\ \vdots \\ a_N b_M \dots a_2 b_M a_1 b_M a_0 b_M \end{array}$$

Multiplier

1 11 111 11
↖ ↖ ↖ ↖
0 0 1 1 ← 3
0 1 1 1 ← 7
1 1 1 1 ← 15
+ 0 0 1 1 ← 3

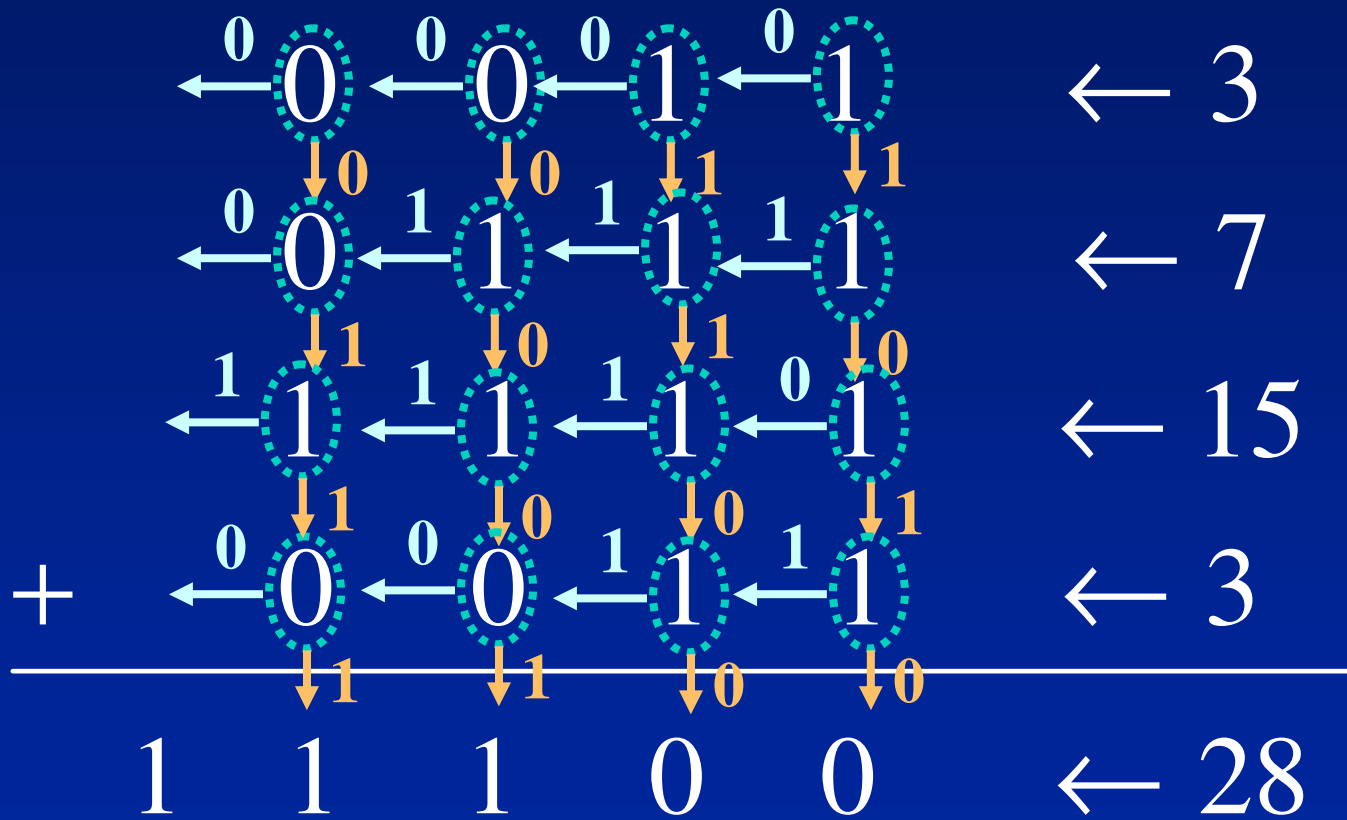
1 1 1 0 0 ← 28

Multiplier

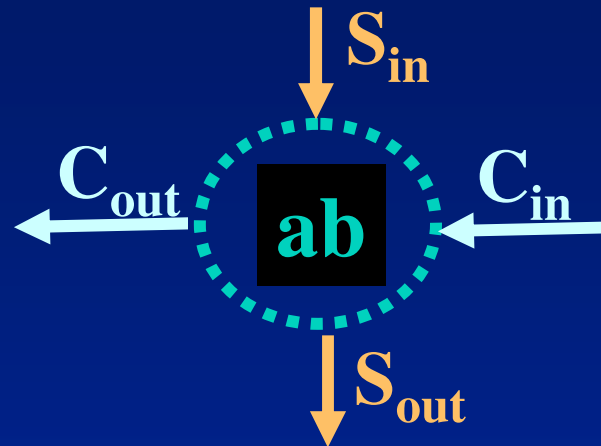
$$\begin{array}{rcccccc} & 0 & 0 & 1 & 1 & & \leftarrow 3 \\ & 0 & 1 & \overset{1}{\leftarrow} \textcircled{1} \overset{1}{\leftarrow} & 1 & & \leftarrow 7 \\ & 1 & 1 & 1 & 1 & 1 & \leftarrow 15 \\ + & 0 & 0 & 1 & 1 & & \leftarrow 3 \\ \hline 1 & 1 & 1 & 0 & 0 & & \leftarrow 28 \end{array}$$

The diagram illustrates a 5-bit multiplier operation. The multiplicand is 0011 (decimal 3) and the multiplier is 0111 (decimal 7). The partial products are 0011, 0111, 1111, and 0011. The sum of these partial products is 11100 (decimal 28). The central '1' in the second row is circled in green, with arrows pointing to it from the '1's in the first, second, and third rows, indicating the carry propagation process.

Multiplier



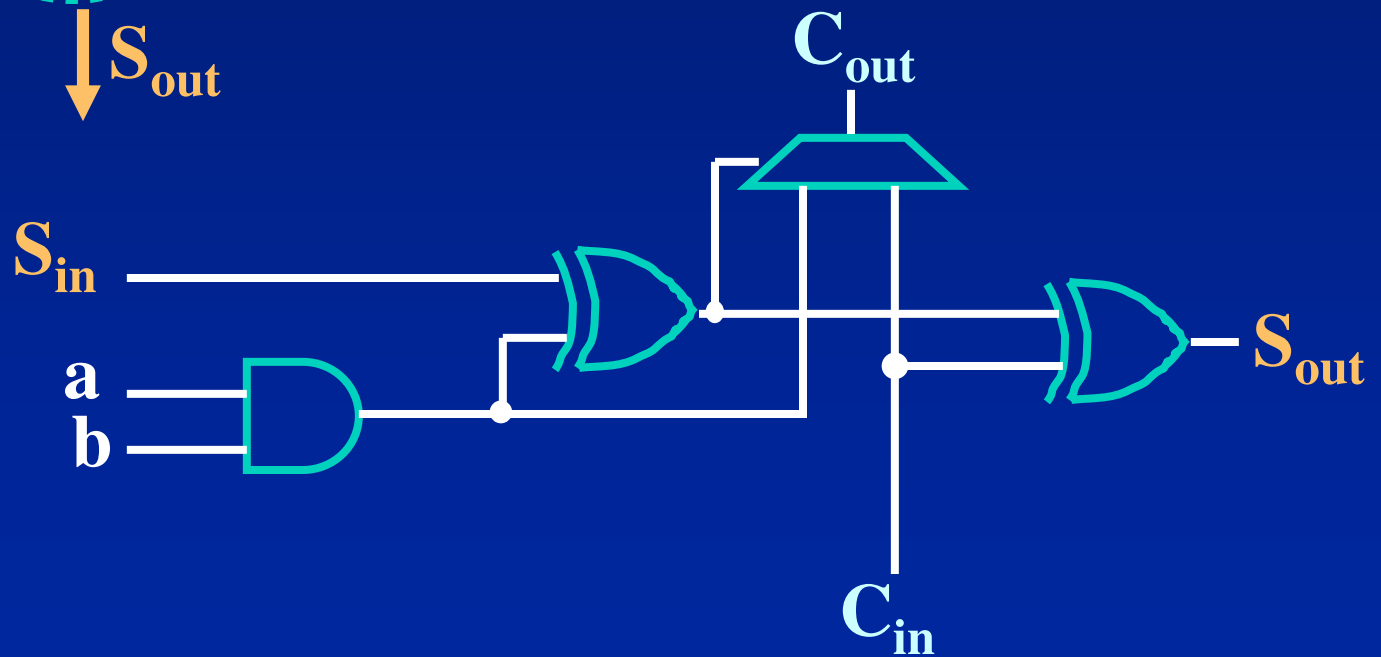
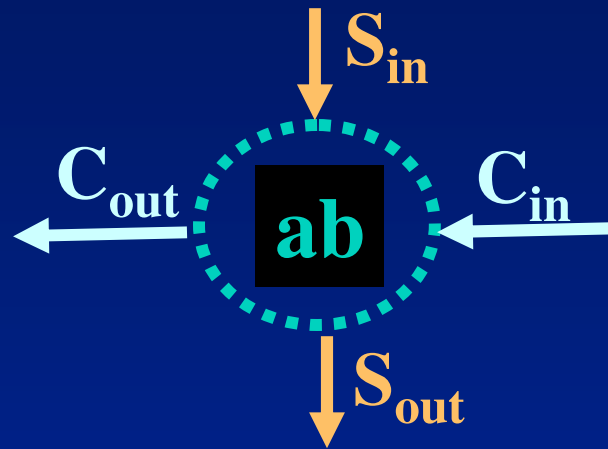
Multiplier



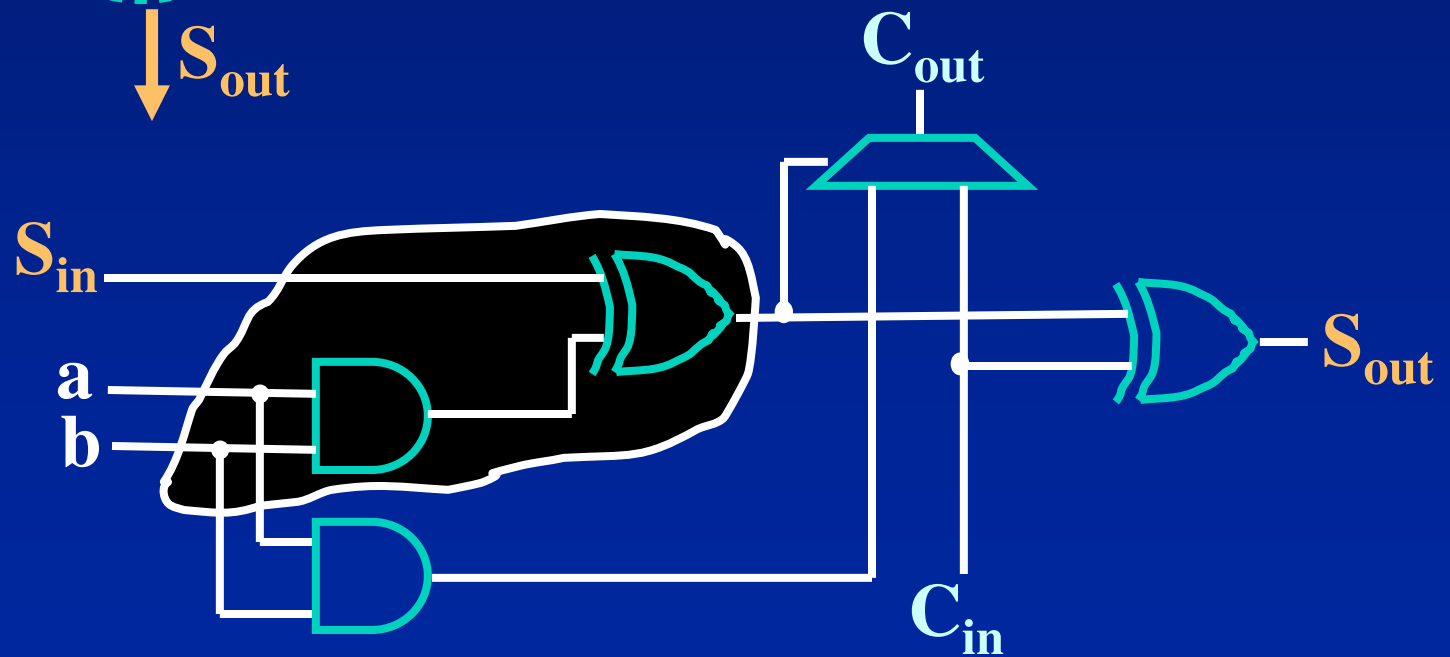
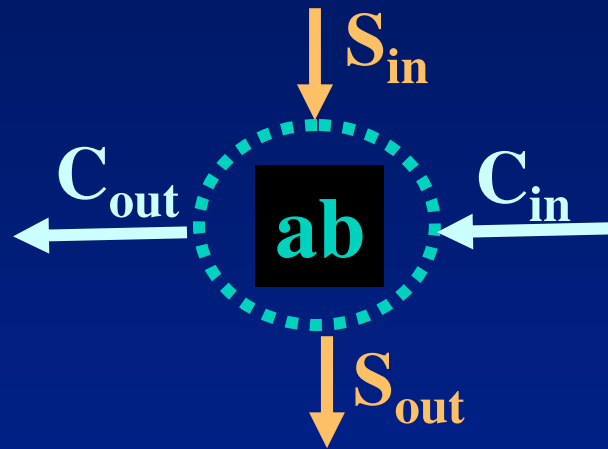
$$S_{out} = (ab \oplus S_{in}) \oplus C_{in}$$

$$C_{out} = abS_{in} + S_{in}C_{in} + abC_{in}$$

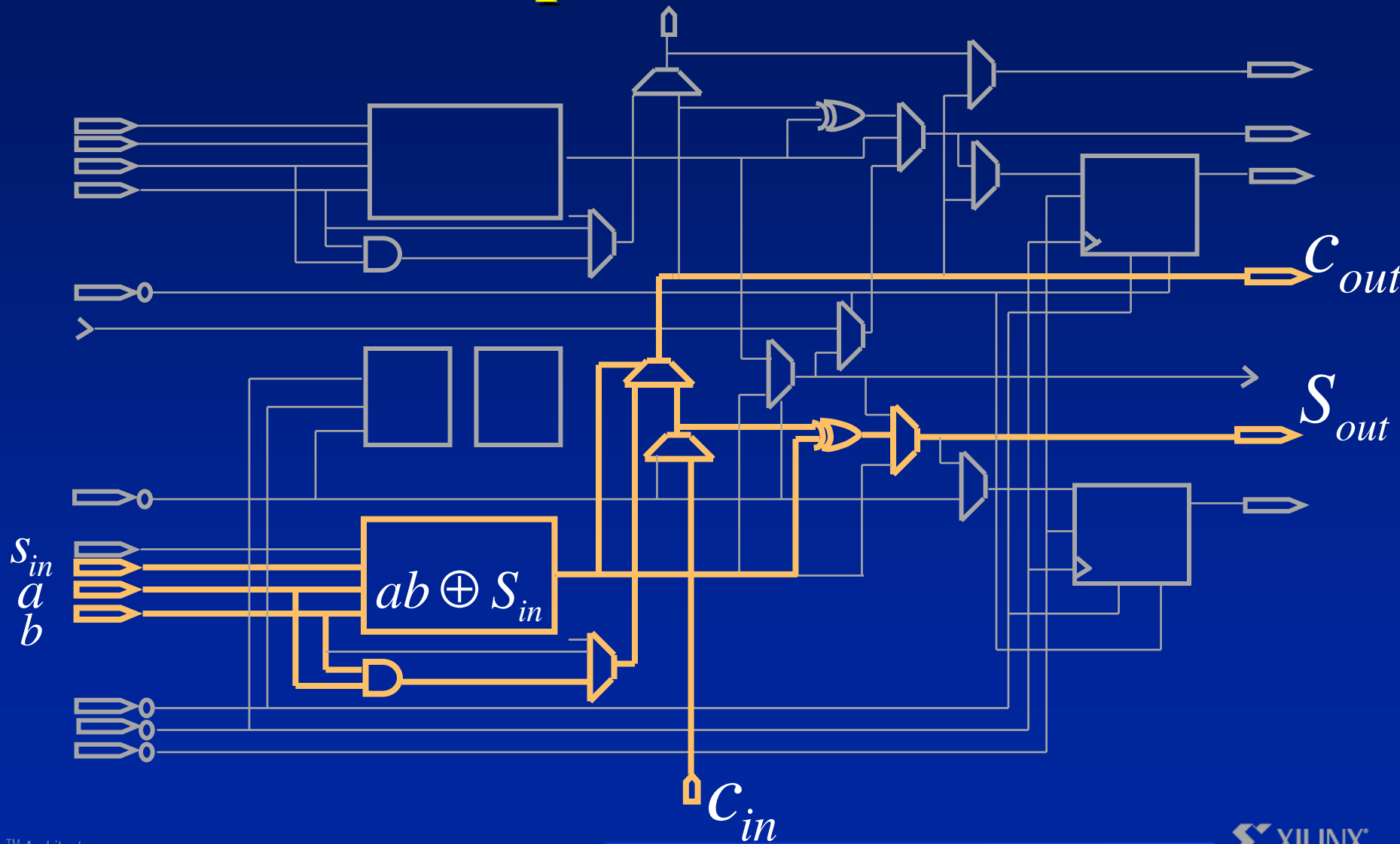
Multiplier



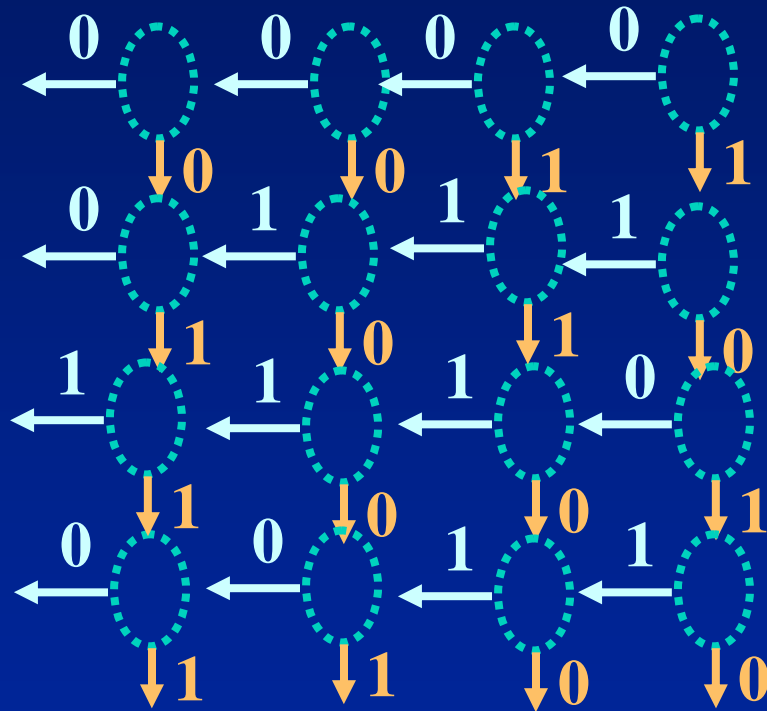
Multiplier



Multiplier core

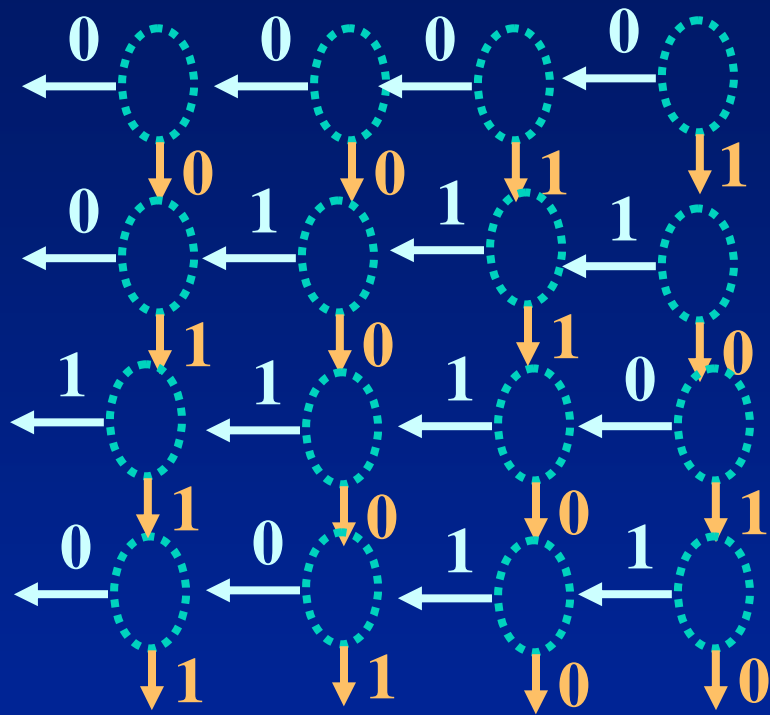


N-bit Multiplier

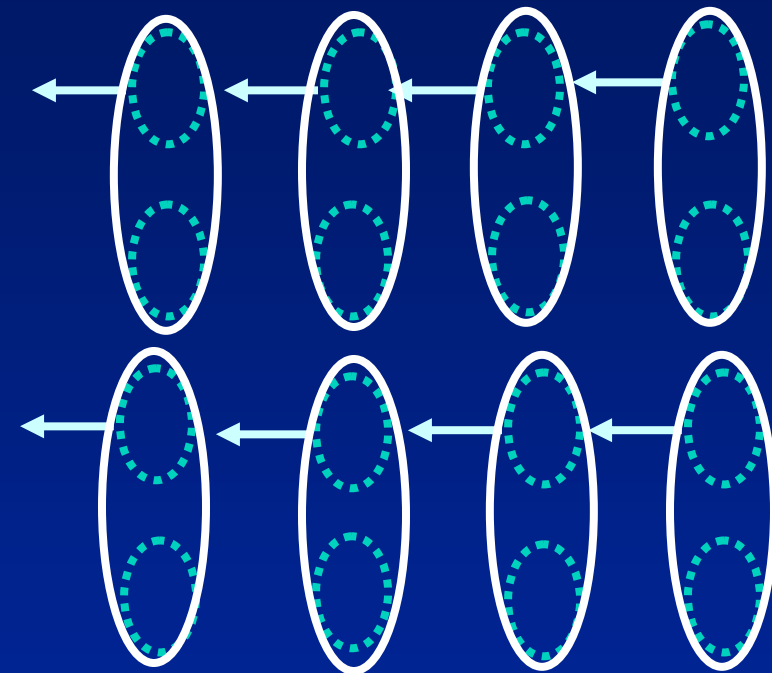


- ◆ Area: N^2 LUTs
= $N^2/2$ slices
= $N^2/4$ CLBs
- ◆ Speed : $2Nc + Ns$
 c = carry delay = 0.1ns
 s = slice delay = 3ns

N-bit Multiplier (efficient)

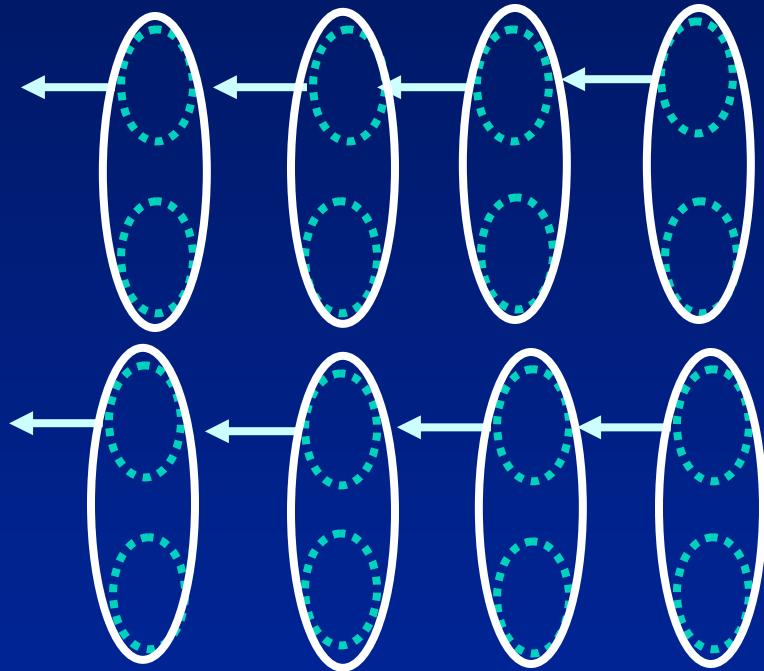




 $ab + S_{in} + c_{in}$

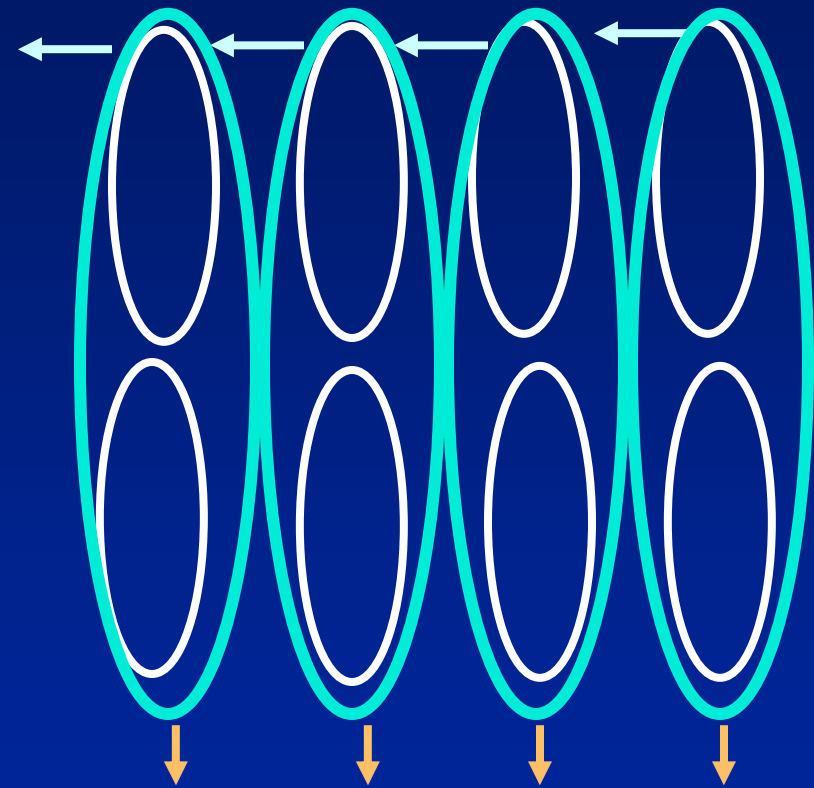



$ab + cd + c_{in}$


N-bit Multiplier (efficient)

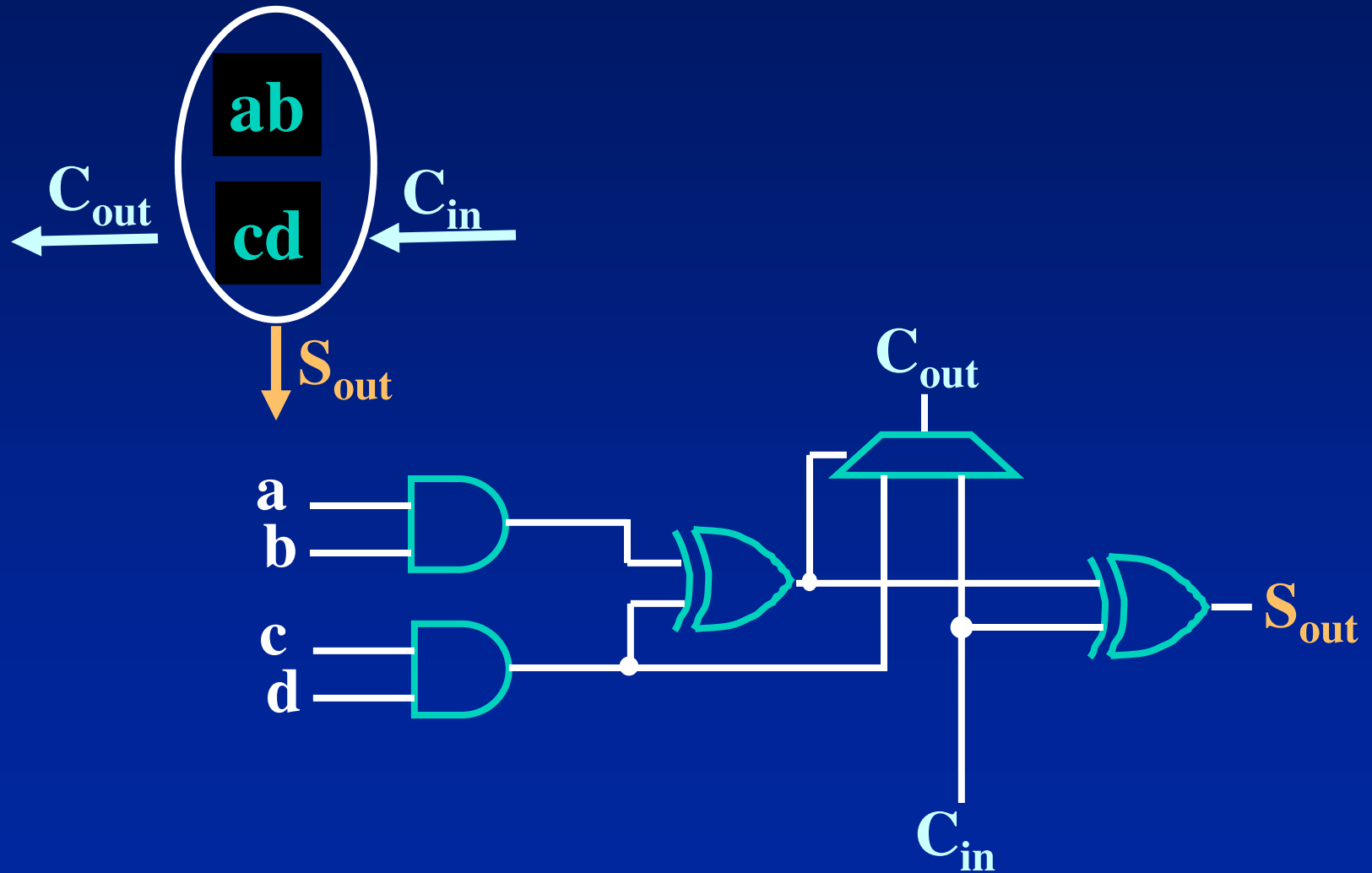


 $ab + cd + c_{in}$

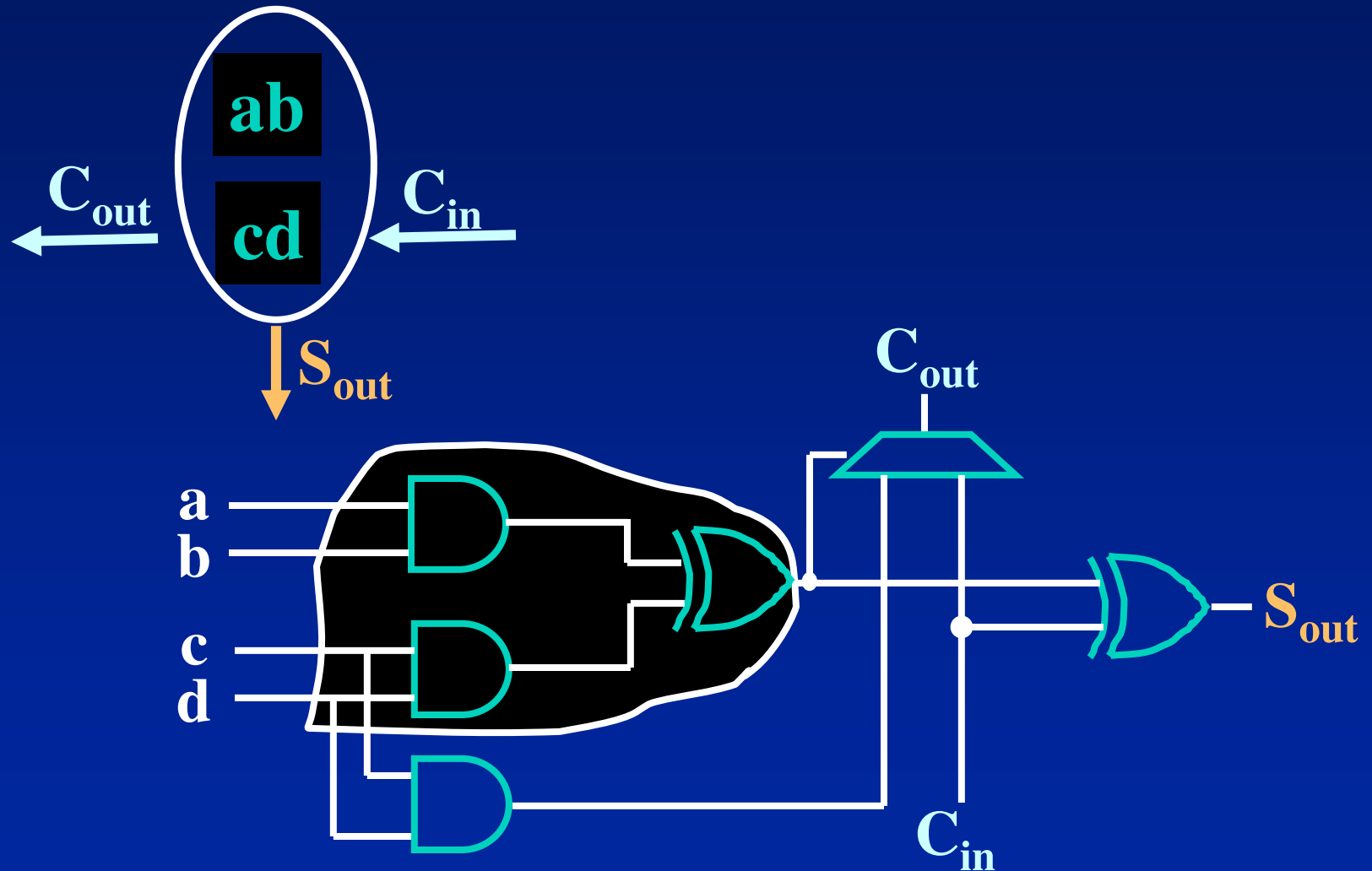


$a + b + c_{in}$ 

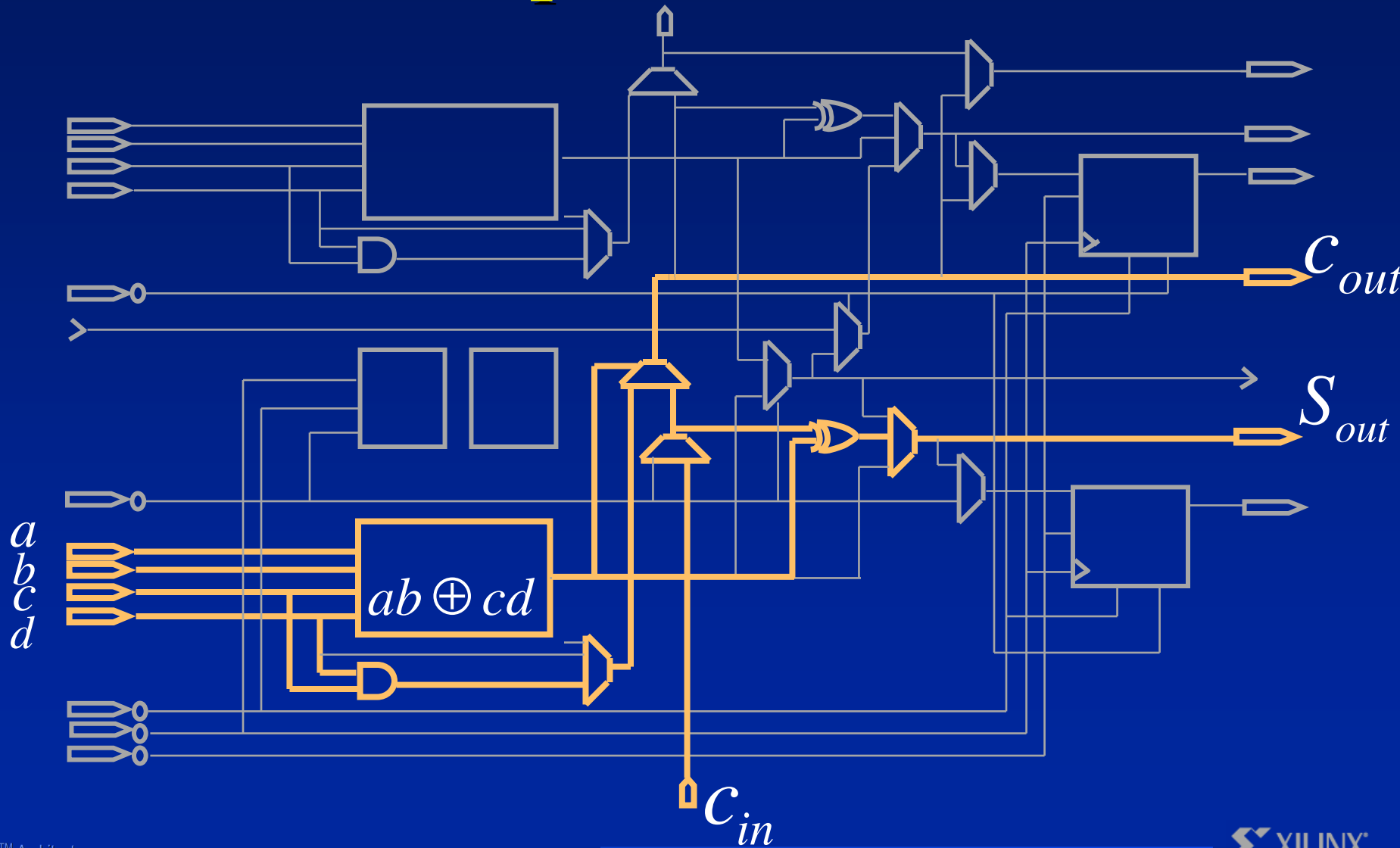
Multiplier (efficient)



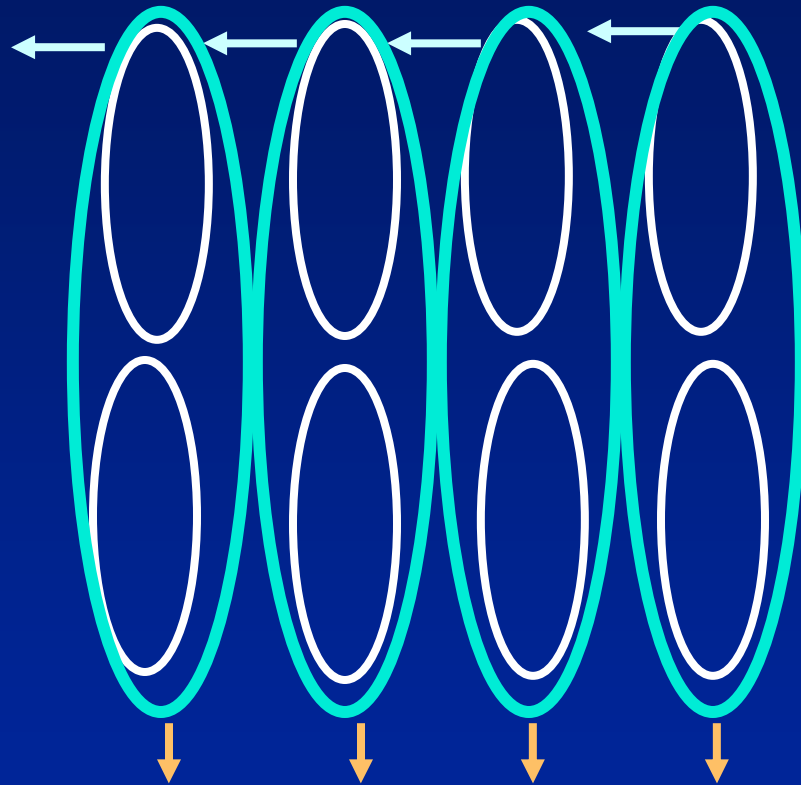
Multiplier(efficient)



Multiplier core



N-bit Multiplier (efficient)



- ◆ Area (LUTs):

$$v = \log_2 N$$

area =

$$\sum_{k=1}^{v-1} N \times \left(\frac{N}{2^k} + 1 \right)$$

- ◆ Speed : $2Nc + v_s$

Constant-Coeff Multiplier

$$Y = k \times X$$

- ◆ K : 8-bit constant coefficient
- ◆ X : 8-bit variable input
- ◆ Y : 16-bit output

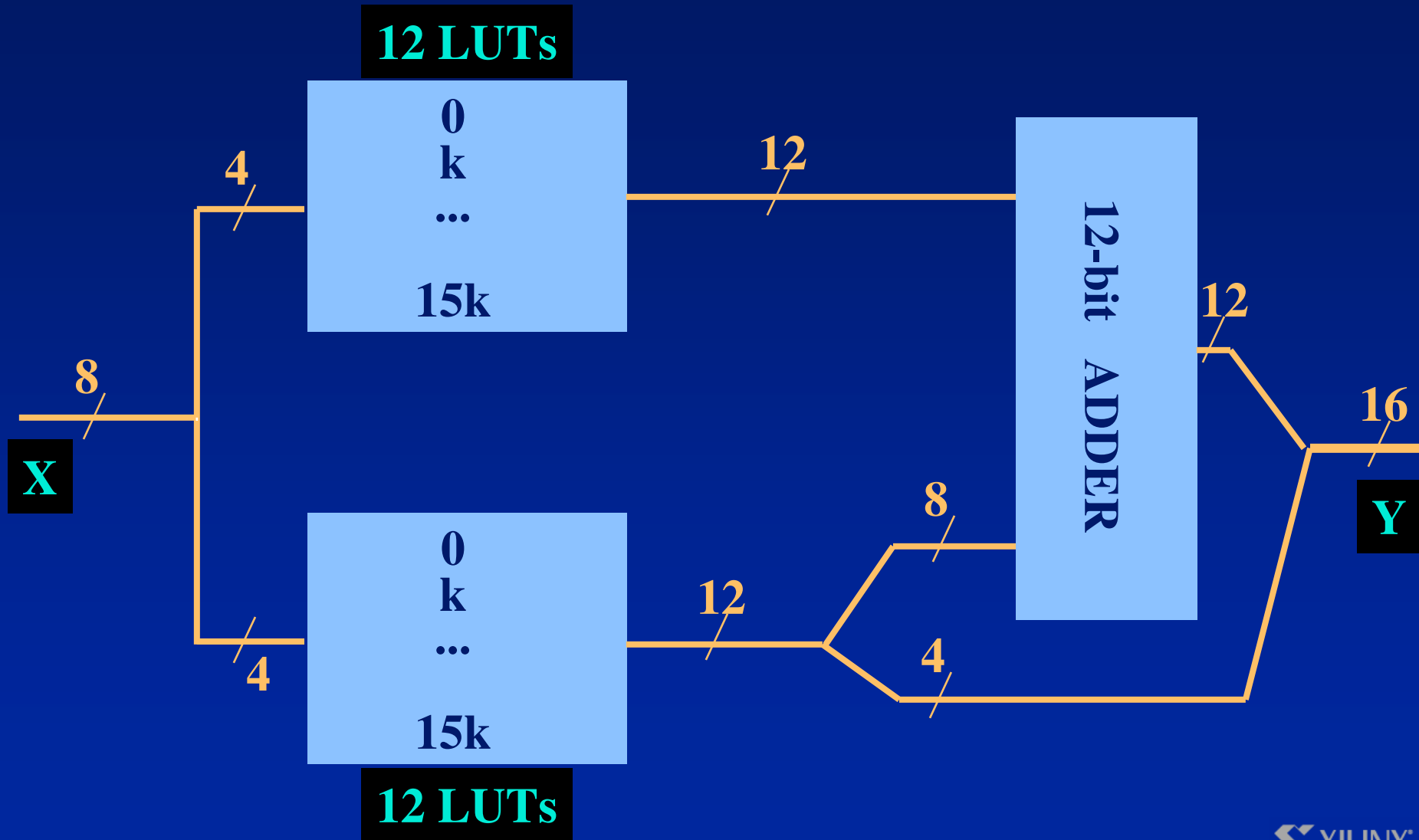
Constant-Coeff Multiplier

$$Y = k \times X$$

- ◆ K : 8-bit constant coefficient
- ◆ X : 8-bit variable input
- ◆ Y : 16-bit output

$$\begin{aligned} Y &= 2^0 k \times X_0 + 2^1 k \times X_1 + \dots + 2^7 k \times X_7 \\ &= \left[2^0 k \times X_0 + \dots + 2^3 k \times X_3 \right] + \\ &\quad \left[2^0 k \times X_4 + \dots + 2^3 k \times X_7 \right] \times 2^4 \end{aligned}$$

Constant-Coeff Multiplier



Const-coeff mult: 8-bits

- ◆ Area:
 - 2 LUTs 12 bit wide \Rightarrow 24 LUTs
 - 1 adder 12 bit wide \Rightarrow 12 LUTs
 - Total area = 36 LUTs
- ◆ Speed:
 - 2s (2 levels of LUT)

Constant-coeff: Booth-style

- ◆ Each run of 1 represents one ADD and one SUB
- ◆ Isolated 1 represents one ADD

$$\begin{aligned} & \mathbf{11011011} \times 01000111 \\ & = \mathbf{11011011}000000 + \\ & \quad \mathbf{11011011}0000 - \\ & \quad \mathbf{11011011} \end{aligned}$$

Constant-coeff: Booth-style

Coeff = 10001011

- ◆ Area
 - A = add shift-1 and shift-2 => 11 LUTs
 - B = add shift-4 and shift-7 => 13 LUTs
 - C = A + B => 16 LUTs
 - Total area = 11+13+16=40 LUTs (> 36 LUTs)
- ◆ Speed
 - 2s or s
- ◆ Better to use KCM

Constant-coeff: Booth multiplier

Coeff = 00011101

- ◆ Area
 - $A = \text{add shift-0 and shift-5} \Rightarrow 14 \text{ LUTs}$
 - $C = A - \text{shift-1} \Rightarrow 15 \text{ LUTs}$
 - Total area = $14+15=29 \text{ LUTs}$ ($< 36 \text{ LUTs}$)
- ◆ Speed
 - $2s$ or s
- ◆ Better to use Booth-style

Distributed Arithmetic

- ◆ Linear time-invariant networks

$$y(n) = \sum_{k=1}^N A_k \cdot x_k(n)$$

$y(n)$ = *response of network at time n*

$x_k(n)$ = k^{th} *input var at time n*

A_k = *weight factor*

Distributed Arithmetic

$$x_k = -x_{k0} + \sum_{b=1}^{B-1} x_{kb} \cdot 2^{-b}$$

$$y = \sum_{k=1}^N A_k \cdot \left[-x_{k0} + \sum_{b=1}^{B-1} x_{kb} \cdot 2^{-b} \right]$$

$$= -\sum_{k=1}^N x_{k0} \cdot A_k + \sum_{k=1}^N \sum_{b=1}^{B-1} x_{kb} \cdot A_k 2^{-b}$$

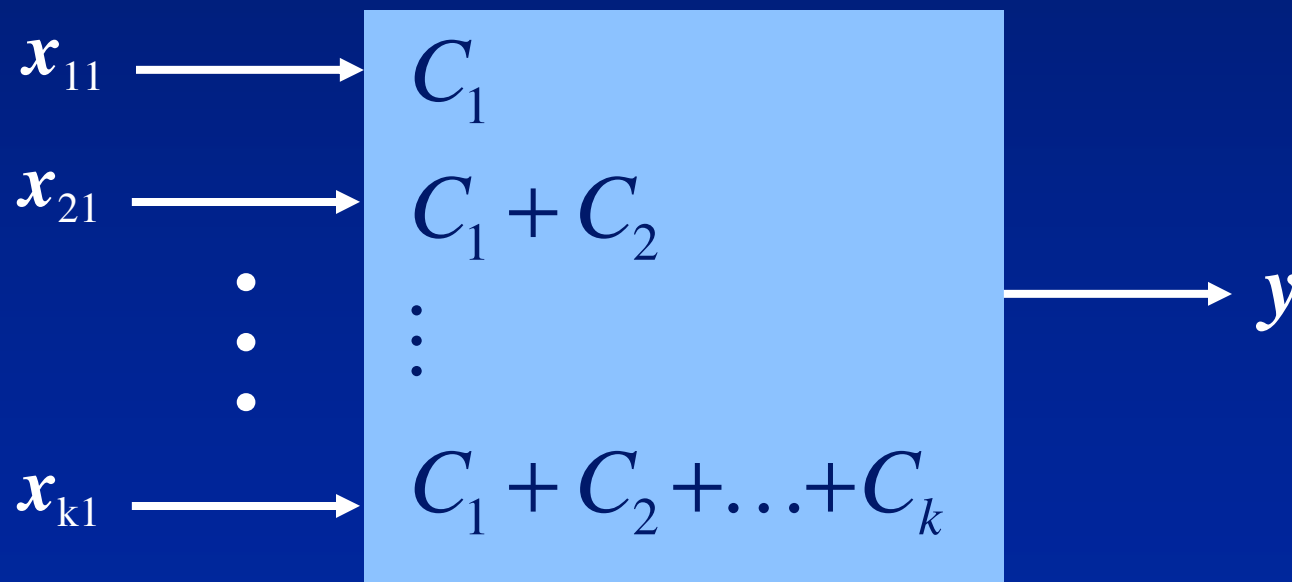
Distributed Arithmetic

- ◆ Expanded form

$$y = - \left[x_{10} \cdot A_1 + x_{20} \cdot A_2 + \dots + x_{k0} \cdot A_k \right] \\ + \left[x_{11} \cdot A_1 + x_{21} \cdot A_2 + \dots + x_{k1} \cdot A_k \right] 2^{-1} \\ \vdots \\ + \left[x_{1(B-1)} \cdot A_1 + x_{2(B-1)} \cdot A_2 + \dots + x_{k(B-1)} \cdot A_k \right] 2^{-(B-1)}$$

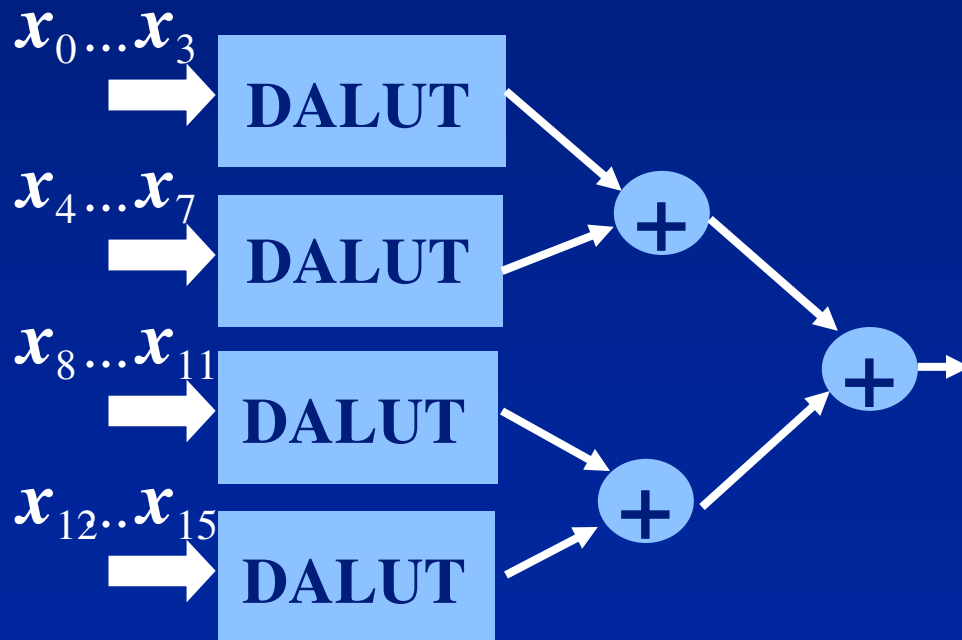
DALUT

$$\left[x_{11} \cdot C_1 + x_{21} \cdot C_2 + \dots + x_{k1} \cdot C_k \right]$$



Distributed Arithmetic

- ◆ $K=16 \Rightarrow 2^{16}$ DALUT contents! $\Rightarrow 512xW$ CLB
- ◆ Break DALUTs to 4/5 inputs and add

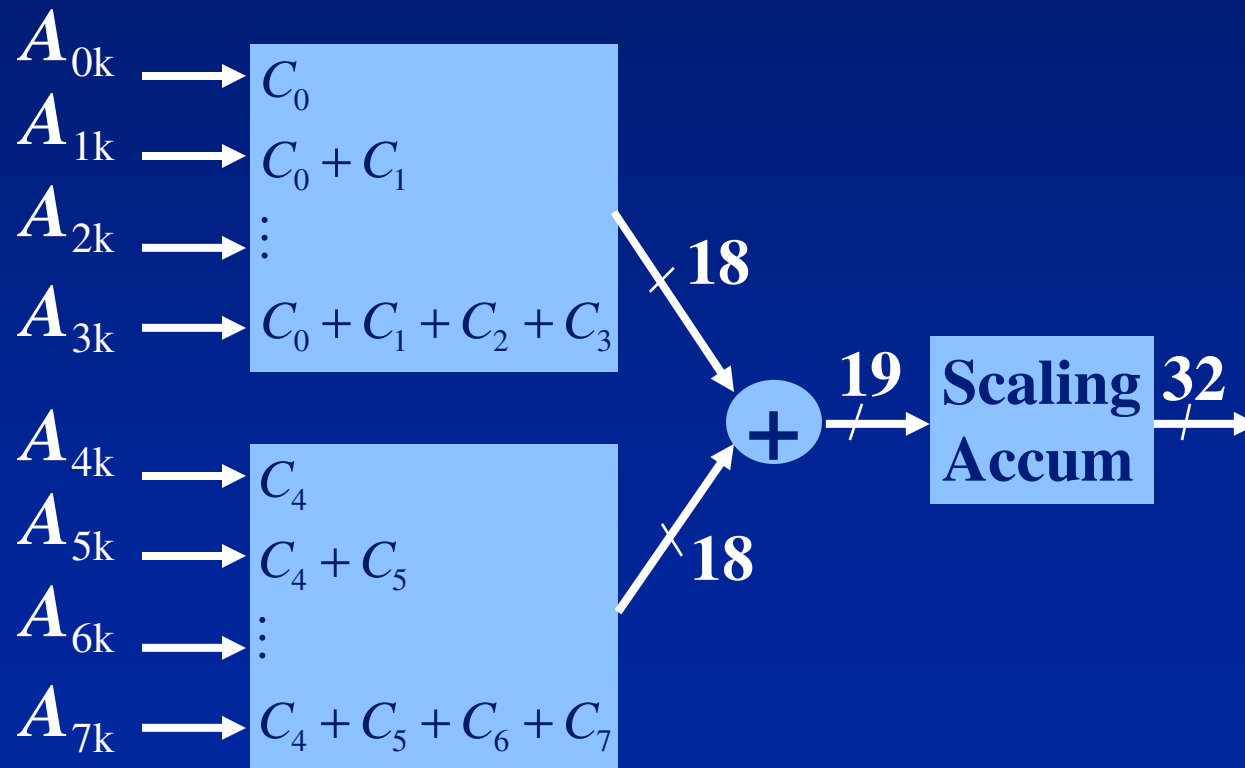


- ◆ Needs W CLBs
- ◆ Delay = $3s$

Area efficient

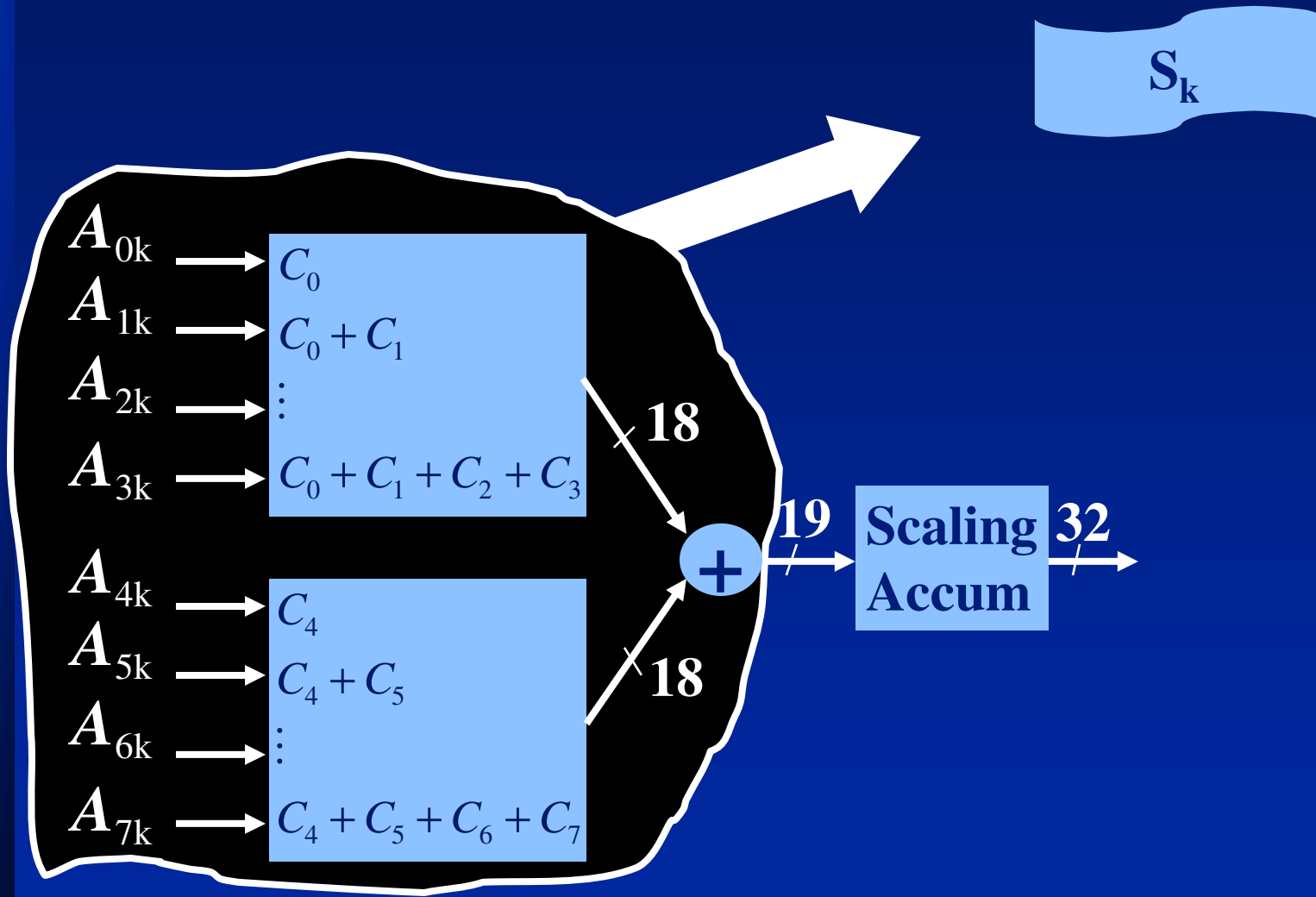
$$Y = C_0 \cdot A_0 + C_1 \cdot A_1 + \dots + C_7 \cdot A_7$$

- ◆ C_k : 16-bit
- ◆ A_k : 16-bit

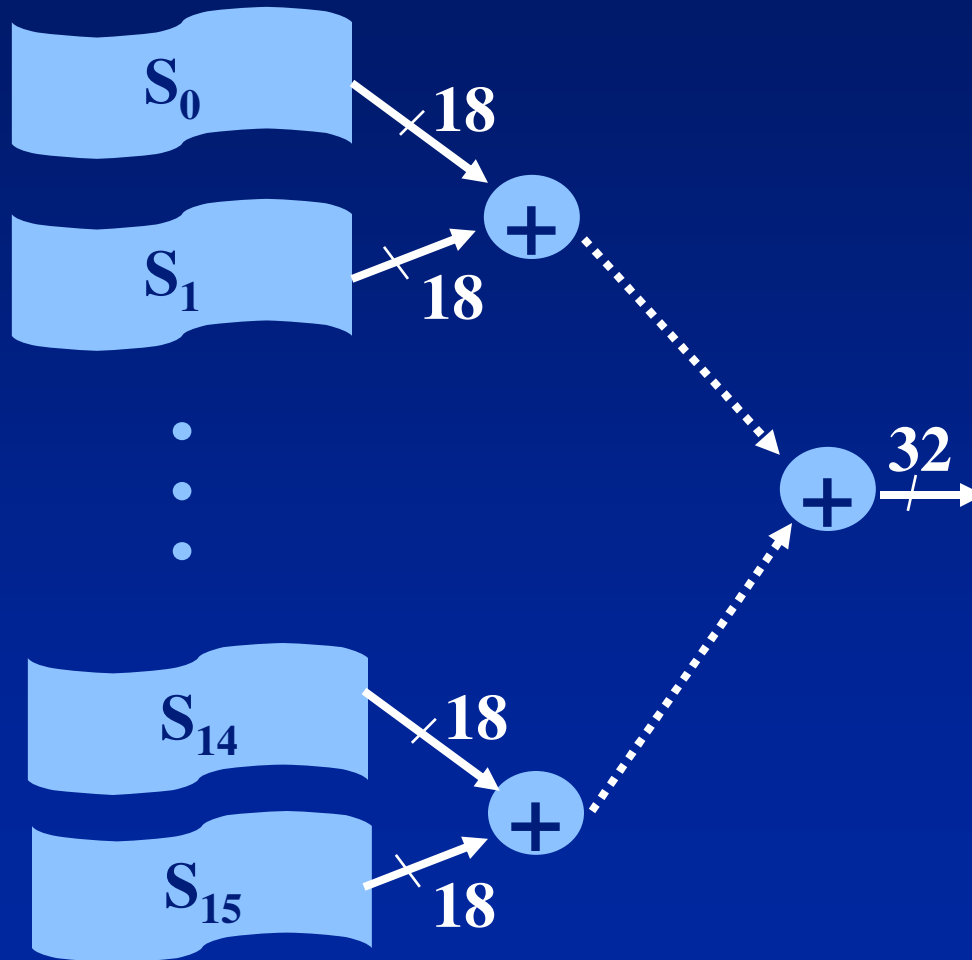


- ◆ Area:
 $[2 \cdot 18] + 19 + 32$
 $= 87$ LUTs
- ◆ Speed:
 $[3s \cdot 16] = 48s$

Speed efficient



Speed efficient



- ◆ Area:
 $[2 \cdot 18 \cdot 16] + [8 \cdot 20] + [4 \cdot 23] + [2 \cdot 28] + [32]$
 $= 916 \text{ LUTs (9X)}$
- ◆ Speed:
 $s + 4s = 5s \text{ (9X)}$
- ◆ Intermediate configurations possible

Conclusions

- ◆ Described the latest generation of Xilinx FPGA: VIRTEX™
- ◆ Architecture is very well-suited for DSP core functions:
 - Adders / Subtractors
 - Wide AND / Ors
 - Comparators / Decoders
 - Multiplexers
 - Multipliers
 - Distributed Arithmetic